TOPOLOGICAL CONDITIONS FOR ELEMENT EVALUATION

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Abstract

In this paper the problem of fault diagnosis is approached from a topological point of view using the two graph representation of networks (the current graph and the voltage graph). A necessary and almost sufficient condition to diagnose multiple faults in active networks is derived. In this condition the concept of a common tree is rigorously employed to extend the validity and usefullness of topological fault analysis for active networks. Based on this condition, which depends only on the graph of the network and not on the element values, the choice of measurement nodes can be substantially simplified.

I. INTRODUCTION

Topological analysis has provided a useful insight to the problem of analog fault diagnosis [1-4]. The topology of the network reveals the connections between different elements aiding the diagnosability and fault location.

One possible approach to network diagnosis is first to assume the fault has occured in a part of the network and then check whether the assumption is correct by examining the consistency of certain linear equations which are invariant on faulty elements. This assume and check method was originally formulated by Biernacki et al. [1]. It was further extended to formulate topological conditions to determine faults in a linear circuit. In [2] Starzyk and Bandler showed a necessary topological condition for the assume and check method with the aid of Coates flow graph representation of a network. Huang et al. [3] introduced the concept of f-node fault testability and they derived necessary and almost sufficient conditions for diagnosability of passive networks.

The main contribution of this paper is to show that a necessary and almost sufficient topological condition for the assume and check method is valid for passive as well as active networks.

This condition is derived on the basis of the two graph representation of active networks, namely, the current graph and the voltage graph. An important feature of this topological condition is that it depends only on the graph representation of the network and not on the element values, which makes it adequate for the diagnosability of analog networks where elements always deviate from their nominal values. It appears that the existence of a common tree is crucial to extend the validity of topological analysis to active networks. The paper is organized as follows. In Section II the algebraic conditions for fault diagnosis are briefly discussed [2-3]. In Section III the proposed topological condition is presented and justified. In Section IV an example is given in which the topological condition is used to determine diagnosability. The effect of this condition on the proper choice of measurement nodes is also investigated.

II. ALGEBRAIC CONDITIONS FOR FAULT DIAGNOSIS

Assume that a network S has n+1 nodes m of them accessible for excitation and measurement with f faulty nodes where $f < m$.

Starting from the nodal equations of the network S we can formulate an overdetermined system of equations. The consistency of this system of equations is a necessary condition to extract the faulty nodes [2-3].

The nodal equations for the nominal values of the elements have the form

$$\underset{\sim}{Y}\,\underset{\sim}{V}_n = \underset{\sim}{I}_n \tag{1}$$

where $\underset{\sim}{Y}$ is the nodal admittance matrix, $\underset{\sim}{V}_n$ is the vector of nodal voltages with respect to a selected reference node g, $\underset{\sim}{I}_n$ is the vector of nodal currents.

If S is perturbed to (S+ΔS) with the same excitations we obtain

$$(\underset{\sim}{Y} + \Delta\underset{\sim}{Y})(\underset{\sim}{V}_n + \Delta\underset{\sim}{V}_n) = \underset{\sim}{I}_n \tag{2}$$

Subtracting (1) from (2) yields

$$\underset{\sim}{Y}\,\Delta\underset{\sim}{V}_n = -\,\Delta\underset{\sim}{Y}\,\underset{\sim}{V}_n{}' = \Delta\underset{\sim}{I}_n \tag{3}$$

where $\underset{\sim}{V}_n{}' = \underset{\sim}{V}_n + \Delta\underset{\sim}{V}_n$ is the vector of nodal voltages and $\Delta\underset{\sim}{I}_n$ represents changes in nodal currents caused by faulty elements. Defining node i as faulty if and only if the ith component of $\Delta\underset{\sim}{I}_n$ is nonzero and assuming that only f elements are faulty we can write (3) as

$$\begin{bmatrix} \Delta V^m \\ \Delta V^{n-m} \end{bmatrix} = Y_n^{-1} \Delta I_n = \begin{bmatrix} Z_{mn} \\ Z_{n-m,n} \end{bmatrix} \Delta I_n \quad (4a)$$

$$= \begin{bmatrix} Z_{mf} & Z_{m,n-f} \\ Z_{n-m,f} & Z_{n-m,n-f} \end{bmatrix} \begin{bmatrix} \Delta I_n^f \\ Q \end{bmatrix} \quad (4b)$$

Thus,

$$\Delta V^m = Z_{mf} \Delta I_n^f \quad (5)$$

where ΔI_n^f is the nonzero part of ΔI_n. For $f < m$ equation (5) represents an overdetermined system of equations. A necessary condition for isolating the faulty nodes is the consistency of this system of equations.

Any f faulty nodes can be uniquely located if the following rank test, known as the f-node fault testability condition is satisfied [3],

$$\text{Rank } Z_{mq} = f+1 \quad (6)$$

for all possible q, where q refers to (f+1) columns of Z_{mn}. However, the above condition is too strong if we are interested in a certain set of faults F.

Considering a specific candidates for faulty nodes F, the following condition can be used [2],

$$\text{Rank } Z_{mx} = f+1 \quad (7)$$

where, in this case, x = card X refers to (f+1) columns of Z_{mn}, $X = F \cup \{y\}$, $y \in N-F$ and N is the set of all nodes excluding the reference node. This condition is equivalent to the existence of a square nonsingular submatrix of Z_{mx} of order (f+1).

III. TOPOLOGICAL CONDITION

In this section we show that the condition stated in (7) can be transformed to a condition which depends only on the graph of the network and not on the element values.

Let $S_m=\{s_1,s_2,\dots,s_m\}$ be a set of integer numbers and let K_u be an u element subset of S_m, i.e., $K_u =\{k_1,k_2,\dots,k_u\}$. By $\overline{K}_u$ we understand the complement of K_u in S_m. Let H be mxn matrix with $m < n$, $H(K_u,J_r)$ be the submatrix of H consisting of the rows and columns corresponding to the integers in the set $K_u \subset S_m$ and $J_r \subset S_n$, respectively. In particular, by $M(K_u,.)$ we understand $M(K_u,S_n)$ and by $M(.,J_r)$ we understand $M(S_m,J_r)$.

Lemma [2]

Let $Z(E,X)$ be a square submatrix of the matrix $Z(M,X)$, then

$$\det Z(E,X) \neq 0 \iff \det Y(\overline{X},\overline{E}) \neq 0 \quad (8a)$$

where $Y=Z^{-1}$ and M is the set of measurement nodes.

We will now investigate the topological implications of the above lemma. A topological criterion for the $\det Y(\overline{X},\overline{E})$ to be nonzero will be presented.

Let G be a graph and $V = \{v_1,v_2,\dots,v_f\}$ a subset of the set of its vertices. Denote t(V) to be f-tree of G such that the vertices of V belong to different connectivity components. T(V) is the set of all possible f-trees t(V). G(V) is the graph obtained from G by the identification of all the vertices of V (i.e., by shorting these vertices to the reference node).

Theorem

A necessary and almost sufficient condition for the matrix $Y(\overline{X},\overline{E})$ to be nonsingular is

$$T(X \cup \{g\}) \cap T(E \cup \{g\}) \neq \emptyset \quad (8b)$$

where g is the reference node.

Proof

Necessity: The nodal admittance matrix of the network can be written as

$$Y = A_i Y_b A_v^T \quad (9)$$

where A_i is the incidence matrix of the current graph G_i of the network, A_v is the incidence matrix of the voltage graph G_v of the network and Y_b is the diagonal branch admittance matrix. Accordingly we may write

$$Y(\overline{X},\overline{E}) = A_i(\overline{X},.) Y_b A_v^T(.,\overline{E}) \quad (10)$$

Removing rows from the incidence matrix of the graph is equivalent to short circuiting the corresponding nodes to the reference node [5]. Hence, $A_i(\overline{X},.)$ is the incidence matrix of the current graph $\hat{G}_i = G_i(X \cup \{g\})$ and $A_v(\overline{E},.)$ is the incidence matrix of the voltage graph $\hat{G}_v = G_v(E \cup \{g\})$. Consequently we denote $A_i(\overline{X},.)= \hat{A}_i$ and $A_v(\overline{E},.) = \hat{A}_v$.

Using Binet-Cauchy theorem [6] the $\det Y(\overline{X},\overline{E})$ can be expanded as follows

$$\det Y(\overline{X},\overline{E}) = \sum_{K_r} \det \hat{A}_i(.,K_r) \det Y_b(K_r,K_r) \det \hat{A}_v^T(K_r,.) \quad (11)$$

where r is the rank of $\hat{G}_i(\hat{G}_v)$.

Since $\det Y_b(K_r,K_r)\neq 0$ we conclude that $\det Y(\overline{X},\overline{E})$ is nonzero only if $\det \hat{A}_i(.,K_r)\neq 0$ and $\det \hat{A}_v^T(K_r,.) \neq 0$ for a certain K_r. Furthermore, if K_r represents a tree of $\hat{G}_i$ then $\det \hat{A}_i(.,K_r)$ is nonzero [5]. The same is true for $\det \hat{A}_v^T(K_r,.)$ in $\hat{G}_v$. Therefore $\det Y(\overline{X},\overline{E})$ is nonzero if and only if the set of branches K_r represents a common tree of both the current graph $\hat{G}_i$ and the voltage graph $\hat{G}_v$. In other words $\det \hat{A}_i(.,K_r) \neq 0$ if and only if K_r represents (f+2)-tree $t(X \cup \{g\})$ in the current graph G_i and similarly $\det \hat{A}_v^T(K_r,.)\neq 0$ if and only if K_r represents (f+2)-tree $t(E \cup \{g\})$ in G_v. Hence $Y(\overline{X},\overline{E})$ is nonsingular only if there exists a common (f+2)-tree of both the current and the voltage graphs with nodes $X \cup \{g\}(E \cup \{g\})$ in different connectivity components of $G_i(G_v)$ respectively.

Sufficiency : The sufficiency follows from the important fact that the edges of the two graph representation of the network (the current graph and the voltage graph) have unique weights [6]. $\det Y(\bar{X},\bar{E})$ is a polynomial of edge weights and we will show that if the specified common tree exists then this determinant is not zero and we do not have a symbolical cancellation.

We will show that the polynomial representing $\det Y(\bar{X},\bar{E})$ is not zero for a particular set of edge weights, therefore on the basis of [7], it is not zero for almost all edge weights. The polynomial representing $\det Y(\bar{X},\bar{E})$ can be written as

$$P = y_{i_1} P_1 + \text{summation of other terms which do not include } y_{i_1} \qquad (12)$$

where we factor out the weight y_{i_1} of a selected edge e_{i_1} and P_1 is the polynomial obtained after this factorization. From (11) and the existance of a common tree at least one term will appear on the right hand side of (12). Now P_1 can also be written as

$$P_1 = y_{i_2} P_2 + \text{summation of other terms which do not include } y_{i_1} \text{ or } y_{i_2} \qquad (13)$$

where y_{i_2} is the weight of a second selected edge and P_2 is the polynomial obtained after factoring out y_{i_1} and y_{i_2} We will now turn our attention to P_2 which can be written as

$$P_2 = y_{i_3} P_3 + \text{summation of other terms which do not include } y_{i_1}, y_{i_2} \text{ or } y_{i_3} \qquad (14)$$

Continuing this way we finally obtain

$$P_{n-1} = y_{i_n} + \text{summation of other terms which do not include } y_{i_1}, y_{i_2}, \ldots, y_{in} \qquad (15)$$

Since y_{i_n} is not zero therefore P_{n-1} can be made not zero by proper selection of y_{i_n} and similarly we can select $y_{i_{n-1}}$ such that P_{n-2} is not zero.

Continuing backwards we can get P_1 not zero and select y_{i_1} which makes the polynomial P not zero. Accordingly $\det Y(\bar{X},\bar{E}) \neq 0$ for the selected values of edge weights $y_{i_1}, y_{i_2}, \ldots, y_{i_n}$ and on the basis of [7] it follows that the determinant is not zero for almost all edge weights.

IV. EXAMPLE

Consider the active network with two voltage controlled current sources shown in Fig. 1.

The topological condition given in the Theorem can be used to decide upon a number and a location of measurement nodes to identify faulty nodes. The current and voltage graphs of the network are shown in Fig. 2. Let the measurement nodes be 1 and 2, thus we have the set $E = \{1,2\}$. The nodal admittance matrix of the network is given by

$$Y = \begin{bmatrix} y_1+y_2 & 0 & -y_2 & 0 \\ 0 & y_4+y_6 & -y_4 & g_2 \\ y_2-g_1 & -y_4 & y_4+y_2+g_1 & 0 \\ g_1 & 0 & -g_1 & y_5 \end{bmatrix}$$

The graph $\hat{G}_v = G_v(E \cup \{5\})$ is obtained from G_v by shorting the nodes 1 and 2 to the reference node (i.e., node 5).

Assume that node 3 is the faulty node, i.e., $F=\{3\}$, thus $y \in N-F = \{1,2,4\}$. If there exists a common tree of both $G_v(E \cup \{4\})$ and $G_i(X \cup \{5\})$ where $X= F \cup \{y\}$, then $Y(\bar{X},\bar{E})$ is nonsingular and therefore we can uniquely identify node 3 as faulty. In other words we are looking for a common 3-tree $t(X \cup \{5\})$ in G_i and $t(E \cup \{5\})$ in G_v.

Consider $X=\{3,4\}$. As we can see from Fig. 3, $y_2 g_2$ is a common tree of both $G_i(X \cup \{5\})$ and $G_v(E \cup \{5\})$, i.e., $y_2 g_2$ represents a common 3-tree of both G_i and G_v. This can be verified by examining the determinant of the matrix $Y(\bar{X},\bar{E})$ which in this case is as follows

$$Y(\bar{X},\bar{E}) = \begin{bmatrix} -y_2 & 0 \\ -y_4 & g_2 \end{bmatrix}$$

The other two possibilities of the set X required to check node 3 as faulty are shown in Fig. 4 and Fig. 5. In Fig. 4 it is easy to see that $y_2 y_5$ is a common tree, i.e., $y_2 y_5$ is a common 3-tree of both G_i and G_v. The corresponding nonsingular matrix $Y(\bar{X},\bar{E})$ is given by

$$Y(\bar{X},\bar{E}) = \begin{bmatrix} -y_2 & 0 \\ -g_1 & y_5 \end{bmatrix}$$

Finally from Fig. 5 we find that $y_4 y_5$ and $g_1 g_2$ are common 3-trees of both G_i and G_v where the corresponding nonsingular matrix $Y(\bar{X},\bar{E})$ is

$$Y(\bar{X},\bar{E}) = \begin{bmatrix} -y_4 & g_2 \\ -g_1 & y_5 \end{bmatrix}$$

In both cases graph $\hat{G}_v$ is as shown in Fig.3b).

Hence the topological condition stated in the Theorem is satisfied and we conclude that the faulty node 3 can be uniquely located provided that 1 and 2 are the measurement nodes.

As another example consider the same network with $E = \{1,4\}$ as the set of measurement nodes, and node 2 as the faulty node, i.e., $F = \{2\}$. We will investigate the existance of a common tree of both $G_i(E \cup \{5\})$ and $G_v(X \cup \{5\})$ where $X = F \cup \{y\}$, $y \in N-F= \{1,3,4\}$.

Consider the case $X = \{2,3\}$ shown in Fig. 6. It is clear that there is no such a common tree, so the matrix $Y(\bar{X},\bar{E})$ has to be singular. This can be checked by inspecting the matrix $Y(\bar{X},\bar{E})$ which in this case is given by

$$Y(\bar{X},\bar{E}) = \begin{bmatrix} 0 & -y_2 \\ 0 & -y_1 \end{bmatrix}$$

Hence, we conclude that it is not possible to identify node 2 as faulty with the measurements chosen at nodes 1 and 4.

V. CONCLUSIONS

A necessary and almost sufficient topological condition for diagnosability in analog circuits has been fully presented and justified. The representation of the network through a pair of graphs (the current graph and the voltage graph) together with the concept of a common tree constitute the theoritical basis of the proposed condition.
A unique feature of this topological condition is that it is applicable to passive as well as active networks. This represents a significant advantage over other topological conditions presented in [2-3]. The implications on the choice of measurement nodes is illustrated through an example.

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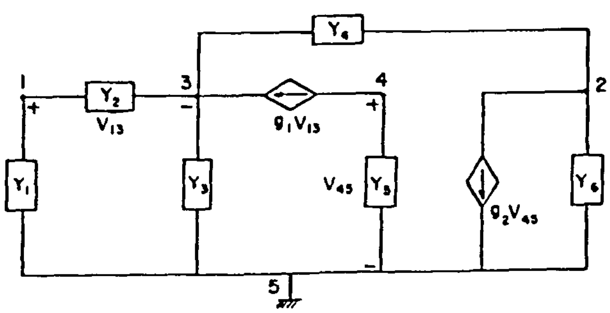

Fig. 1 Active network.

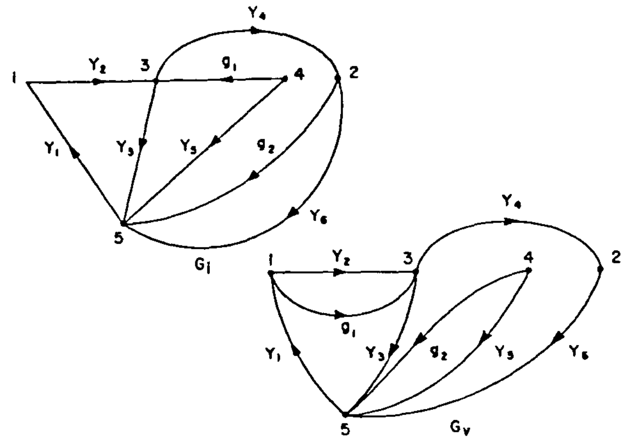

Fig. 2 The current and the voltage graphs.

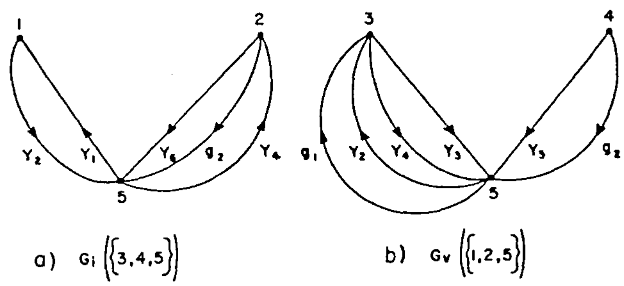

Fig. 3 The current and the voltage graphs.

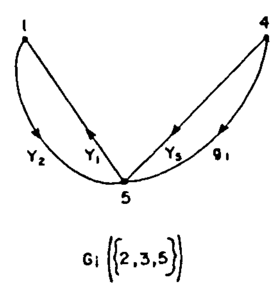

Fig. 4

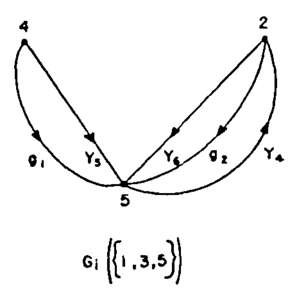

Fig. 5

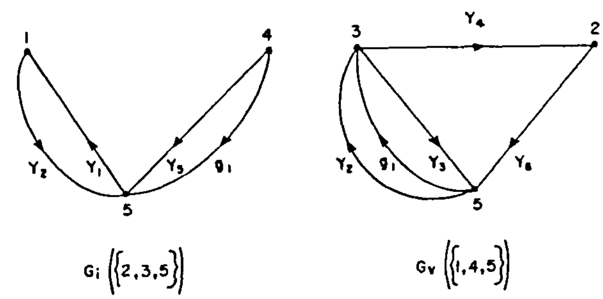

Fig. 6 The current and the voltage graphs.