TIME DOMAIN TESTING OF LARGE NONLINEAR CIRCUITS

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1. Introduction

VLSI fabrication of large analog or integrated analog and digital circuits has become both cost effective and practical. Artificial neural networks built using VLSI technology [1] push testing requirements of large scale analog circuits beyond the limits of existing methods. As the complexity of analog integrated circuits increases, the need for automatic tests of these circuits becomes a critical requirement in circuit fabrication and maintenance. In complex systems it is not only more difficult to access different subsystems for functional testing, but also more difficult to analyze the test results since the computations increase with the cube of the number of network elements. Testing is also an economic issue as test costs become the major expense in circuit fabrication. Thus the major objective for automatic testing of large scale, analog circuits is to perform reliable tests with minimum test cost and computational effort.

Testing of analog and mixed (analog and digital) networks is a challenging and difficult task [2]. In spite of excellent theoretical and practical methods developed over the years [3]–[5], analog testing lags behind digital testing in both the size and complexity of circuits that can be practically diagnosed. This limitation is closely tied to similar limitations of analog circuit simulators [6]. On the other hand, sensitivities, which play a key role in circuit testing, can be evaluated at a minimum cost if they are calculated together with the solution vector [7]. So, if the circuit simulation can be performed more efficiently, then the sensitivity evaluation and fault diagnosis will be greatly facilitated.

This paper presents a new approach to analog and mixed mode testing based on a decomposition technique. Voltage measurements placed at the partition points are used to reduce the effect of a faulty element to a local area, thus facilitating the test. Limiting the effect of a fault to a local area allows the separation of digital and analog parts as their analyses do not have to be performed simultaneously. In the proposed testing technique measurements play an active role not only on the assessment of circuit functionality but on the circuit simulation as well. This active role of voltage measurements changes the way we simulate the circuit and improves the speed and accuracy of the diagnosis process.

Any practical testing method must consider finite accuracy of computer simulation as well as effects of measurement errors on the validity of the results obtained. Therefore, to estimate element testability we use numerical rank evaluation of the test matrix based on the singular value decomposition [8]. The QR factorization approach [9] is used to select the best set of test points. In time domain testing, the test points are test nodes, sampling time points, DC excitation levels, and types of input waveforms. The test point selection minimizes prediction standard deviations or estimation errors resulting from random measurement errors.

In order to derive the basis for the decomposition approach and to compare it with the existing methods, we briefly describe the sensitivity approach. The sensitivity approach is the most popular testing method to this day. It can handle a broad category of circuits and testing situations. But it shows serious drawbacks when it is applied to large circuits: 1) it needs large computation time and memory space, 2) it is sensitive to errors caused by the circuit model, numerical methods, and measurements, 3) it cannot be directly applied to the mixed mode circuits. We introduce the decomposition approach to eliminate these drawbacks.

In this paper, we present the description of our method for the time domain testing. The method can be extended for the measurements of harmonic components of the periodic response. We start discussion on the nonlinear system equations and sensitivity approach, then introduce the decomposition approach. The test procedure of the proposed approach is given.

2. Nonlinear System Equations

Consider a nonlinear circuit described by the set of algebraic differential equations in implicit form:

$$f(\dot{x}, x, p, t) = 0 \ , \tag{1}$$

where

f is the vector of circuit functions,
x is the vector of circuit variables,
$\dot{x}$ is the vector of time derivatives of x,
p is the vector of circuit parameters, and
t is time.

It is assumed that at t=0, the initial conditions are consistent and (1) has a unique solution. The system equations (1) can be obtained using any general formulation technique such as the modified nodal formulation, sparse tableau or hybrid descriptions.

The time interval $(0,\tau)$ is divided using discrete time points $(0,t_1,t_2,\ldots,\tau)$. At each time point, the solution of (1) is first determined by a nonlinear solver. Then the sensitivity of (1) w.r.t. all parameters can be obtained simultaneously with the solution vector, by solving a linear equation.

System solution

At a certain time point t_j, (1) becomes a nonlinear algebraic equation

$$f(\dot{x}_j, x_j, p) = 0 \ . \tag{2}$$

In order to solve (2) by the Newton–Raphson method or other iterative techniques, the Jacobian matrix M_j is evaluated by

$$M_j = \frac{df}{dx_j} = \frac{\partial f}{\partial \dot{x}_j}\frac{\partial \dot{x}_j}{\partial x_j} + \frac{\partial f}{\partial x_j} \ . \tag{3}$$

Using the backward differentiation formula (BDF), we can formulate

$$\dot{x}_j = \sum_{\ell=0}^{k_j} a_\ell x_{j-\ell} \tag{4}$$

where

$$h_j = t_j - t_{j-i} \ . \tag{5}$$

and

$$a_\ell = \frac{\alpha_{\ell j}}{h_j} \tag{6}$$

From (4) we get

$$\frac{\partial \dot{x}_j}{\partial x_j} = a_0 \ , \tag{7}$$

and the Jacobian matrix (3) becomes

$$M_j = \frac{\partial f}{\partial \dot{x}_j} a_0 + \frac{\partial f}{\partial x_j} . \qquad (8)$$

Derivatives $\partial f/\partial \dot{x}_j$ and $\partial f/\partial x_j$ are calculated using current values of the solution vector x. Since these values change from iteration to iteration, therefore the Jacobian matrix M_j has to be evaluated and factorized at each iteration. The nonlinear iterations are solved using

$$M_j \Delta x_j = - f_j , \qquad (9)$$

where

$$f_j = f(\dot{x}_j, x_j, p) . \qquad (10)$$

If the iterations (9) are performed using the nominal parameter values p^0, then a nominal solution vector x^0 is obtained.

Sensitivity Approach

In our analysis, we assume that the system parameters p are close to their nominal values. The purpose of the fault diagnosis is to find deviations $\Delta p = p - p^0$, which characterize changes in the element equations. Linear elements are described through their admittances; therefore only one parameter is required to identify each linear element. Nonlinear elements have their characteristics described through several parameters p_i (e.g. $i_b = p_0 \exp(p_1 v_b) + p_2$), so one nonlinear element may require identification of more than one value in order to determine its characteristics.

To obtain the sensitivity of the original system (2) w.r.t. parameter p, differentiate both sides of (2) which yields

$$\frac{\partial f}{\partial \dot{x}_j} \frac{\partial \dot{x}_j}{\partial p} + \frac{\partial f}{\partial x_j} \frac{\partial x_j}{\partial p} + \frac{\partial f}{\partial p} = 0 . \qquad (11)$$

Let us denote

$$s_j = \frac{\partial x_j}{\partial p} , \qquad (12)$$

then (11) becomes

$$\frac{\partial f}{\partial \dot{x}_j} \dot{s}_j + \frac{\partial f}{\partial x_j} s_j = - \frac{\partial f}{\partial p} . \qquad (13)$$

Using BDF, we can write $\dot{s}_j$ as

$$\dot{s}_j = a_0 s_j + \sum_{\ell=1}^{k_j} a_\ell s_{j-i} . \qquad (14)$$

After some manipulations, (11) becomes

$$\left[\frac{\partial f}{\partial \dot{x}_j} a_0 + \frac{\partial f}{\partial x_j}\right] s_j = - \frac{\partial f}{\partial p} - \frac{\partial f}{\partial \dot{x}_j} \sum_{\ell=1}^{k_j} a_\ell s_{j-i} \qquad (15)$$

System of linear equations (15) has the form:

$$M_j s_j = - B_j \qquad (16)$$

where

$$B_j = \frac{\partial f}{\partial p} + \frac{\partial f}{\partial \dot{x}_j} \sum_{\ell=1}^{k_j} a_\ell s_{j-\ell}$$

After the sensitivity matrix S at all time points (t_1, t_2, ..., t_j, ...,τ) is formulated, the element deviations Δp can be obtained by solving the test equations

$$S \Delta p = \Delta v \qquad (17)$$

where Δv are the deviations of the measured responses from the nominal response.

It is clear from the above analysis that the sensitivities can be obtained by solving a linear equation (15) after the solution of the original system has been obtained. Therefore, solution of the original system is the most time consuming step and is also the most inaccurate step in sensitivity evaluation. Inaccuracy of the system model, approximations of nonlinear integration, and solution of nonlinear algebraic equations introduce errors to the sensitivity analysis. In the next section a decomposition approach is described which significantly reduces these deficiencies. As a result, analog testing strategies can be developed and implemented for networks many times larger than those which can be handled by existing methods.

3. Decomposition Approach

It is well known that decomposition approaches to network analysis are very effective in reducing the amount of computation when the size of the analyzed network becomes large. The decomposition approach proposed in [10] for fault location in large-scale networks is using the fault verification technique. In this case it is assumed that the faulty elements are located within a small part of the network and the remaining part is fault-free. Decomposition of a network into smaller subnetworks facilitate testing by localizing the effect of faults.

In this paper, we apply the decomposition approach to solve fault identification and element evaluation problems. We seek savings both in evaluation of the solution vector x and the vector of parameter deviations Δp. Our goal is realized in two steps: network analysis and network testing.

Network Analysis

Let N be the network under test. The nodal decomposition decomposes the network N into k subnetworks by hypothetically breaking the connections (not actually cutting connecting wires) at accessible nodes (see Fig. 1). There must be no mutual coupling between any two subnetworks. We assume that all decomposition nodes can be accessed for measurements. The measured nodes are the external nodes (denoted by m) and the remaining nodes are internal nodes (denoted by i).

After the voltage measurements have been taken at the external nodes of each subnetwork, the external variables x^m have known values. In this case, the deviation of measured voltages Δx^m are zero

$$\Delta x^m = 0 \qquad (18)$$

and since the measured voltages do not vary with the assumed or computed parameter values, sensitivities of the external variables to the parameters are zero,

$$s^m = \frac{\partial f^m}{\partial p} = 0 . \qquad (19)$$

Therefore, the system solution vector x^i and the sensitivities s^i can be easily computed. When the circuit is decomposed into a number of small subnetworks, all computations for x^i and s^i can be implemented in parallel within each subnetwork.

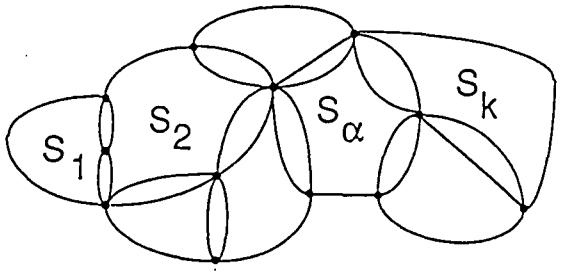


Fig. 1. Subnetwork of N.

Internal System Solutions

The deviations of circuit variables only contain the internal part

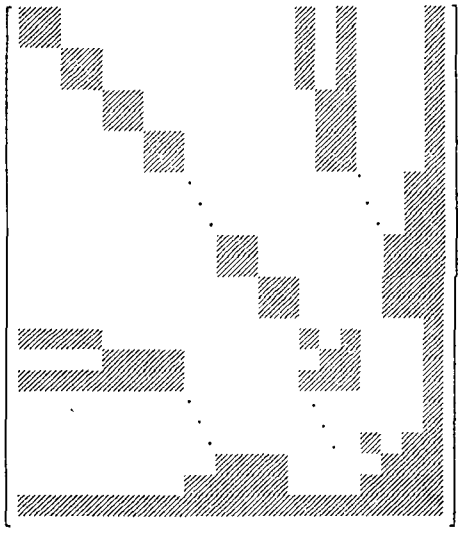
$$\Delta x = \begin{bmatrix} \Delta x^i \\ \Delta x^m \end{bmatrix} = \begin{bmatrix} \Delta x^i \\ 0 \end{bmatrix}, \qquad (20)$$

so only internal system equations f^i are used for solving internal variables Δx^i. As a result we can simplify (9), eliminating rows and columns of M_j which correspond to the measured circuit variables and denoting the obtained submatrix by M_j^i. Then (9) can be replaced by

$$M_j^i \, \Delta x_j^i = - f_j^i, \qquad (21)$$

and the internal variables are evaluated using the measured variables and the nominal parameter values.

To observe the effect of this simplification, consider the structure of M_j and M_j^i. If the network can be partitioned hierarchically at the measurement nodes, then M_j has the following structure:

$M_j =$ 

where the block diagonal part of M_j corresponds to the internal circuit variables. Therefore the submatrix M_j^i is block diagonal and (21) becomes

$$\begin{bmatrix} M_j^{i_1} & & & & \\ & M_j^{i_2} & & & \\ & & \ddots & & \\ & & & M_j^{i_\alpha} & \\ & & & & M_j^{i_k} \end{bmatrix} \begin{bmatrix} \Delta x^{i_1} \\ \Delta x^{i_2} \\ \cdots \\ \Delta x^{i_\alpha} \\ \cdots \\ \Delta x^{i_k} \end{bmatrix}_j = - \begin{bmatrix} f^{i_1} \\ f^{i_2} \\ \cdots \\ f^{i_\alpha} \\ \cdots \\ f^{i_k} \end{bmatrix}_j \qquad (22)$$

(22) can be solved independently in each subnetwork. For $\alpha = 1, 2, \ldots, k$,

$$\boxed{M_j^{i_\alpha} \, \Delta x_j^{i_\alpha} = - f_j^{i_\alpha}}, \qquad (23)$$

The internal variables $x_j^{i_\alpha}$ are evaluated using the measured variables $x_j^{m_\alpha}$ and the nominal parameter values p^α of each subnetwork. Therefore, the internal variables x_j^i can be computed in parallel. A solution vector obtained from (21) is different than the nominal solution x_j^0 and we denote it by $\bar{x}_j$.

When the iterative process converges, the KCL equations are satisfied at the internal nodes, i.e.

$$f_j^i(\bar{x}_j, \dot{\bar{x}}_j, p^0) = 0 \qquad (24)$$

Internal Sensitivities

The internal sensitivities should be calculated before evaluating test matrix. Differentiating both sides of (24) w.r.t. p, we obtain

$$\frac{\partial f_j^i}{\partial \dot{x}_j^i} \frac{\partial \dot{x}_j^i}{\partial p} + \frac{\partial f_j^i}{\partial x_j^i} \frac{\partial x_j^i}{\partial p} + \frac{\partial f_j^i}{\partial p} = 0 \qquad (25)$$

or

$$\frac{\partial f_j^i}{\partial \dot{x}_j^i} \sum_{\ell=0}^{k_j} a_\ell \frac{\partial x_{j-\ell}^i}{\partial p} + \frac{\partial f_j^i}{\partial x_j^i} \frac{\partial x_j^i}{\partial p} + \frac{\partial f_j^i}{\partial p} = 0 \qquad (26)$$

Derivatives are calculated using $x_j = \bar{x}_j$ and $p = p^0$.

Denote $s_j^i = \partial x_j^i / \partial p_j$, then (26) can be rewritten as

$$M_j^i \, s_j^i = - B_j^i, \qquad (27)$$

where

$$B_j^i = \frac{\partial f_j^i}{\partial p} + \frac{\partial f_j^i}{\partial \dot{x}_j^i} \sum_{\ell=1}^{k_j} a_\ell \frac{\partial x_{j-\ell}^i}{\partial p}. \qquad (28)$$

Since the matrix M_j^i has block diagonal structure, (27) can be solved in each subnetwork independently to obtain sensitivity vector s_j^i. For $\alpha = 1, 2, \ldots, k$,

$$\boxed{M_j^{i_\alpha} \, s_j^{i_\alpha} = - B_j^{i_\alpha}},$$

Network Testing

After the network analysis stage, the system equations (2) will be satisfied at the internal points, but in general they will not be satisfied at the partition points. This results from a mismatch between assumed (in our case nominal) parameters p^0 for which iterations (21) were performed, and the actual parameters p^* for which the measurements were taken. We define the external system functions as test functions

$$f_j^m = f_j^m(\dot{x}_j, x_j, p) \quad . \qquad (30)$$

Differentiating f_j^m w.r.t. p, and using the solution of (16) we can now formulate the test equations

$$\frac{d f_j^m}{d p} \Delta p = - f_j^m \qquad (31)$$

from which Δp can be evaluated.

The coefficient matrix $\frac{d f_j^m}{d p}$ is called a test matrix T and it is evaluated by differentiating (30) w.r.t. p,

$$T_j = \frac{\partial f_j^m}{\partial \dot{x}_j^i} \sum_{\ell=1}^{k_j} a_\ell \frac{\partial x_{j-\ell}^i}{\partial p} + \frac{\partial f_j^m}{\partial x_j^i} \frac{\partial x_j^i}{\partial p} + \frac{\partial f_j^m}{\partial p} \quad (32)$$

Denote

$$\dot{C}_j = \frac{\partial f_j^m}{\partial \dot{x}_j^i}, \qquad C_j = \frac{\partial f_j^m}{\partial x_j^i} \text{ and } f_{pj}^m = \frac{\partial f_j^m}{\partial p}, \quad (33)$$

then (32) becomes

$$T_j = \dot{C}_j \sum_{\ell=1}^{k_j} a_\ell s_{j-\ell}^i + C_j s_j^i + f_{pj}^m \quad (34)$$

where the internal sensitivities s_j^i has been computed from (27). The test matrix of each subnetwork can be evaluated in parallel. For $\alpha = 1, 2, ..., k$,

$$\boxed{T_j^\alpha = \dot{C}_j^\alpha \sum_{\ell=1}^{k_j} a_\ell s_{j-\ell}^{i\alpha} + C_j^\alpha s_j^{i\alpha} + f_{p_\alpha j}^{m\alpha}} \quad (35)$$

The test equation at the time instance j will be

$$T_j \Delta p = -f_j^m \quad (36)$$

Procedure to Generate and Solve Test Equations

Test equations in time domain can be formulated and solved using the following procedure:

1. Decompose the tested circuit N into k subnetworks.
2. Perform measurements at the partition nodes to obtain x^m within the time interval $(0-\tau)$.
3. Assume the initial values p^0, and set j=0.
4. Assume the initial values of internal variables x_j^i, and set j=j+1.
5. For each subnetwork do the following steps in parallel (α=1,2,...,k)
   - Predict $x_j^{i\alpha}$ using the forward differentiation formula.
   - Calculate $\dot{x}_j^{i\alpha}$ using the backward differentiation formula.
   - Estimate internal variables $x^{i\alpha}$ by iterative process (23).
   - Compute the sensitivities $s^{i\alpha}$ by solving (29).
   - Evaluate the test matrices T_j^α using (35).
6. Formulate the test matrix T_j by combining test matrices T_j^α of all subnetworks.
7. Repeat Steps 4–6 for each time point within $(0,\tau)$. The test matrix has the form
   $$T = [T_1', T_2', ..., T_\tau']',$$
   where ' stands the matrix transpose operation.
8. Estimate parameter deviations Δp by solving the test equations of the interconnected system
   $$T \Delta p = -f^m.$$
   The test matrix T has bordered block diagonal structure. The sparse matrix technique, parallel algorithm and vector computation can be used to reduce the computational time and the memory requirements.
9. Update the parameter values and repeat steps 3–8 if Δp is large.

Remarks

1. The Jacobian matrix M_j^i for the nonlinear iterations (21) is block diagonal, so the circuit analysis is much easier than that in the sensitivity approach (9).
2. Each subnetwork can be analyzed independently, therefore, parallel processing can be implemented which further reduce the analysis time.
3. In the special case of a linear subnetwork, a solution vector can be obtained in one step; no iterations are necessary. Note that if a linear subnetwork is a part of a nonlinear network, such a simplification of analysis cannot be achieved by other approaches. Even in the popular harmonic balance approach [11], in which linear subnetworks are separated from a nonlinear part, several iterations are necessary to balance the mismatch between the solutions of nonlinear and linear parts.
4. The Jacobian matrix $M_j^{i\alpha}$ is block diagonal with the LU factorization known from the solution of (23), so the sensitivities $s_j^{i\alpha}$ can be easily obtained from (29).
5. It is obvious from the block diagonal form of M_j^i that the internal variables x_j^i of a subnetwork depend only on these parameters from the vector p which belong to the same subnetwork. Also, one can observe that derivatives of f_j^m w.r.t. x_j^i or $\dot{x}_j^i$ are nonzero only for these variables or their derivatives which are in the subnetworks incident at a selected measurement nodes. As a result, the test matrix T_j (32) has a block matrix structure. Due to this structure it is possible to identify individual parameters locally using only measurements from a given subnetwork or two adjacent subnetworks.
6. The Newton–Raphson method requires the Jacobian to be constructed and factorized at each iteration. If the changes in the Jacobian from iteration to iteration are sufficiently small, then the old Jacobian closely approximates the new one. Therefore, the factorized Jacobian from one iteration can be used for several subsequent iterations.

4. Test Procedure

The test procedure, which implements the proposed testing method, will be organized as follows:

1. In the pre–test stage, a circuit under test is modeled in order to perform a computer simulation and test nodes are selected at the partition points.
2. During the actual test, reference time domain input signals are applied to the circuits under test using different signal levels. A waveform recorder is used to sample, digitize and store the output responses at different test points. Data are collected either directly through the voltage probes or indirectly through special purpose testing circuitry (as discussed in [3]). After collection, the data are transferred to the computer system for post–test processing. The time domain testing system is illustrated in Fig. 2. A similar system will be used in frequency domain testing where instead of time samples, different harmonic components of the periodic response will be recorded.
3. In the post–test stage, the measured voltages are used to aid circuit analysis and to formulate the test equations, from which deviations of the network parameters Δp are evaluated. If a mixed analog–digital circuit is tested, then analog signals at the terminals of digital subcircuits are transformed into the digital format and digital testing is performed on these subcircuits. Note that this approach requires the measurements to be performed at the boundaries between the analog and the digital parts of a circuit.

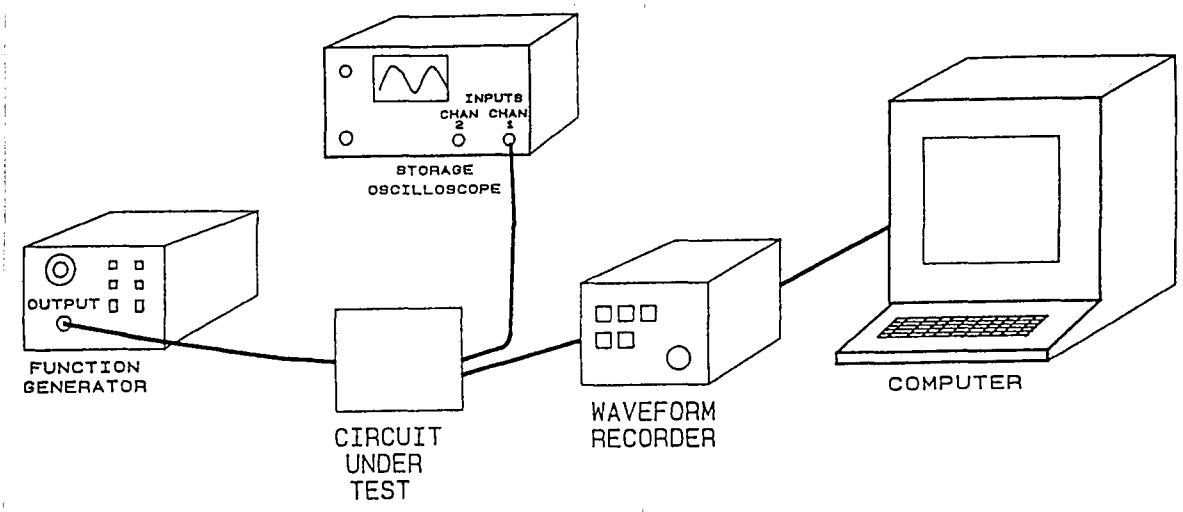


Fig. 2. Time domain testing system.

5. Summary

In the described testing method the measurements are used in an efficient, innovative way to achieve two major goals:
1. To improve circuit simulation.
2. To aid parameter identification.

The first goal is satisfied by breaking interconnected system equations into a set of smaller subsystems. Each of the subsystems can be analyzed separately reducing overall analysis time and memory requirements. In addition, the subsystems can be analyzed using analysis methods which best suit the type of a subcircuit analyzed. For example, a linear subcircuit can be analyzed using the Fourier transform, or another frequency domain method, which takes advantage of the circuit linearity. A subcircuit with only resistive elements may use an algebraic equation solver since the differential equations will not be needed. Other types of subcircuits such as subcircuits with distributed parameters or subcircuits with ideal switches may use specialized analysis methods.

The second goal is satisfied by the same principle of partitioning applied to the test equations. The Jacobian matrix of the test equations has a block matrix structure which permits solution of the parameter identification problem locally. As a result, savings in computer time and memory are realized. Another important effect of the Jacobian matrix structure is limitation of effects of changes in the system to the local areas. Only the parameters of the subnetworks adjacent to a particular test node will affect the test equation at this node.

In addition to the above mentioned advantages, the proposed method allows a more flexible approach to testing different parts of a network. Different subnetworks can be simulated and tested on different levels of circuit representation, like discrete element level, gate level, functional level *etc.*. Some subnetworks may be tested on the functional or the macromodel level, while for others parameter identification on the element level may be performed.

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References

1. J. Hutchinson, Ch. Koch, J. Luo, and C. Mead, "Computing motion using analog and binary resistive networks", IEEE Computer, March 1988, pp. 52–63.

2. K. D. Wagner and T. W. Williams, "Design for testability of mixed signal integrated circuits", Proc. 1988 International Test Conference, (Washington, DC), pp. 823–828.

3. J. W. Bandler and A. E. Salama, "Fault diagnosis of analog circuits," Proc. IEEE, vol. 73, 1985, pp. 1279–1325.

4. N. Sen and R. Saeks, "A measure of testability and its application to test point selection – Theory," Proc. 20th Midwest Symp. Circuits and Systems (Lubbock, TX), 1977, pp. 576–583.

5. V. Visvanathan and A.Sangiovanni–Vincentelli, "A computational approach for the diagnosability of dynamical circuits," IEEE Trans. Computer–Aided Des., vol. CAD–3, 1984, pp. 165–171.

6. G. Casinovi and A. Sangiovanni–Vincentelli, "A new Aggregation technique for the solution of large systems of algebraic equations," IEEE Trans. Computer–Aided Des., vol. CAD–7, 1988, pp. 976–986.

7. W. Nye, D. Riley, A. Sangiovanni–Vincentelli, and A.L.Tits, "DELIGHT.SPICE: An optimization – based system for the design of integrated circuits," IEEE Trans. Computer – Aided Des., vol. CAD–7, 1988, pp. 501–519.

8. G. H. Golub, V. Klema, and G. W. Stewart, "Rank degeneracy and least squares problems," Tech. Rep. TR–456, Computer Science Dept., Stanford University, CA, 1976.

9. G. N. Stenbakken and T. M. Souders, "Test point selection and testability measures via QR Factorization of Linear Models," IEEE Trans. Instrum. Meas., vol. IM–36, no. 2, June 1987, pp. 406–410.

10. A.E.Salama, J.A.Starzyk and J.W.Bandler, "A Unified decomposition approach for fault location in large analog circuits," IEEE Trans. Circuits Syst., vol. CAS–31, 1984, pp.609–622.

11. K.S.Kurdert and A.Sangiovanni–Vincentelli, "Simulation of nonlinear circuits in the frequency domain," IEEE Trans. Computer–Aided Des., vol. CAD–5, pp.521–535, 1986.