E.: CTD'99

# Testing of analog dynamic systems based on integral sensitivity


Volodymyr BRYGILLEWICZ[1] Jacek WOJCIECHOWSKI[2] Janusz STARZYK[3]



### Abstract

The paper introduces a unified methodology of testing analog dynamic multiphenomena systems, i.e. systems composed of blocks exploiting various physical phenomena. System models are developed. Diagnosis algorithm based on integral sensitivity, originally developed for electronic circuits, extended to cover multiphenomena systems.


## 1 Introduction

The paper investigates simulation and testing of multiphenomena systems, i.e. systems that are either homogenous or mixed, with blocks exploiting various physical phenomena. Mechatronic and microelectro-mechanics systems (MEMS) are examples of such objects of growing importance.

The steps of diagnosis of multiphenomena systems are: development of system models and their computer simulation, measurements of modules under test, development of the best suited parameter extraction method based on measurement results and computer simulation.

We consider soft faults in analog, possibly nonlinear systems. The diagnosis algorithm for electronic circuits in the presence of noise [1,2] seems to be a useful starting point, as it is not directly related to the physical nature of the tested object.

## 2 Description of multiphenomena systems

Let us consider a multiphenomena system under test (MSUT). For simplicity sake, we assume that it contains only electronic and mechanical subsystems. As mainly state variables are measurable in mechanical systems we will use state equations to describe the mechanical part:

$$\dot{\mathbf{z}}(t,\mathbf{p}_l)=\mathbf{A}(\mathbf{p}_l)\mathbf{z}(t,\mathbf{p}_l)+\mathbf{B}(\mathbf{p}_l)\mathbf{r}(t). \qquad (1)$$

$\mathbf{z}$ - vector of state variables, $\mathbf{r}(t)$ - vector of reference (excitation) signals. The number of mechanical parameters to be diagnosed (vector $\mathbf{p}_l$) is $l_p$. State variables are mostly inaccessible to testing in electronic circuits, and modified nodal equations are used:

$$\mathbf{C}(\mathbf{P}_n)\,\dot{\mathbf{V}}\,(t,\mathbf{P}_n)+\mathbf{G}\,(\mathbf{P}_n)\,\mathbf{V}\,(t,\mathbf{P}_n)-\mathbf{J}\,(t)=\mathbf{0}. \qquad (2)$$

$\mathbf{V}$ is the vector of nodal voltages and selected branch currents, $\mathbf{G}$, $\mathbf{C}$ - matrices of resistive and reactive elements, $\mathbf{J}$ - excitation vector. The number of electronic diagnosable parameters (vector $\mathbf{P}_n$) is $n_p$. In the sequel we consider linear equations coupling the mechanical and electronic subsystems, but this formulation can be extended to cover nonlinear case as well:

$$[\mathbf{L}_1,\mathbf{L}_2]\begin{bmatrix}\dot{\mathbf{V}}\\ \dot{\mathbf{z}}\end{bmatrix}+[\mathbf{K}_1,\mathbf{K}_2]\begin{bmatrix}\mathbf{V}\\ \mathbf{z}\end{bmatrix}=\mathbf{g}(t), \qquad (3)$$

$\mathbf{L}_1,\mathbf{L}_2$ and $\mathbf{K}_1,\mathbf{K}_2$ are matrices relating electrical and mechanical variables. Vectors of unknowns and of diagnosable parameters of the whole system are: $\mathbf{V}_m=[\mathbf{V}^T,\mathbf{z}^T]^T$, $\mathbf{P}_m=[\mathbf{P}_n^T,\mathbf{P}_l^T]^T$. Combining (1), (2) and (3), we can write equations of linear multiphenomena system:

$$\mathbf{C}_m\,\dot{\mathbf{V}}_m(t)+\mathbf{G}_m\,\mathbf{V}_m(t)-\mathbf{J}_m(t)=\mathbf{0}, \qquad (4)$$

where

$$\mathbf{C}_m=\begin{bmatrix}\mathbf{C} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\\ \mathbf{L}_1 & \mathbf{L}_2\end{bmatrix},\quad \mathbf{G}_m=\begin{bmatrix}\mathbf{G} & 0\\ 0 & -\mathbf{A}\\ \mathbf{K}_1 & \mathbf{K}_2\end{bmatrix},\quad \mathbf{J}_m(t)=\begin{bmatrix}\mathbf{J}(t)\\ \mathbf{Br}(t)\\ \mathbf{g}(t)\end{bmatrix}.$$

Formulation (4) can be extended to cover also nonlinear systems.

**Example 1.** Let us consider an electromechanical transducer [3] (Fig.1) translating current $i(t)$ to rotational movement of rotor ($\varphi,\omega$ are the angular


[1] Department of Theoretical Eng. Fundamentals, University of Lviv, Ukraine.

[2] Institute of Radioelectronics Warsaw University of Technol., Nowowiejska 15/19, 00-665 Warsaw, Poland.

[3] School of Electrical Engineering & Computer Science, Athens, Ohio, USA.

position and rotation velocity of the rotor). Friction damping of the rotor movement is represented by the constant $D$ and the friction force is $D\omega$. The rotor movement is restrained by the spring with constant $K_s$, developing torque $F_s = K_s\varphi$. Electrical torque is $F_e = K_e i$ and voltage induced in the coil: $V_j = K_m\omega$. The equivalent circuit model of the transducers is in Fig. 1b. Two controlled sources couple the electrical and mechanical parts.

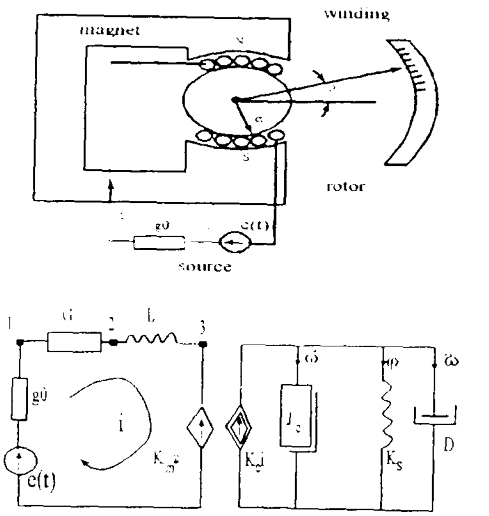



**Fig. 1.** Rotational transducer: a) physical model; b) equivalent circuit model

Modified nodal equations for the electrical part have the form of (2) with:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L \end{bmatrix}, \qquad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ i \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} G+g0 & -G & 0 & 0 \\ -G & G & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} e(t)g0 \\ 0 \\ 0 \end{bmatrix}.$$

The state variables for the mechanical part are $\omega$, $\varphi$ and the describing equations have the form of (1) with:

$$\mathbf{A} = \begin{bmatrix} -\dfrac{D}{J_e} & -\dfrac{K_s}{J_e} \\ 1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \dfrac{1}{J_e} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \omega \\ \varphi \end{bmatrix},$$

$$\mathbf{r}(t) = \begin{bmatrix} K_e I(t) \\ 0 \end{bmatrix}.$$

$J_e$ is the polar inertia moment of the coil assembly. The matrices of coupling equation (3) have the form:

$$\mathbf{L}_1 = \mathbf{0},\ \mathbf{L}_2 = \mathbf{0},\ \mathbf{K}_1 = \begin{bmatrix} 0 & 0 & 0 & K_e \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$$\mathbf{K}_2 = \begin{bmatrix} 0 & 0 \\ K_m & 0 \end{bmatrix}, \qquad \mathbf{g}(t) = \begin{bmatrix} F_e(t) \\ 0 \end{bmatrix}.$$

## 3 Method of diagnosis

System can be accessed at certain points. In electronic circuits by "points" we usually mean nodes. We divide the nodes of an electronic subsystem into three groups: accessible (i.e. at which we apply excitations and measure responses), partially accessible (we only measure responses) and not accessible. The numbers of the nodes in these groups are $n_{acc}$, $n_{pac}$, $n_{ina}$. We divide the points of the mechanical part into $l_{acc}$ accessible "nodes" (where excitation, e.g. force, energy, is applied and responses measured), $l_{pac}$ partly accessible "nodes", (output values, e.g. position, velocity, acceleration, can be measured), and $l_{ina}$ inaccessible "nodes". We can perform total $n_{acc} + l_{acc}$ tests and for each of the tests obtain $n_{acc} + l_{acc} + n_{pac} + l_{pac}$ measurements.

We extend the diagnosis method developed for electronic circuits [1,2] to cover multiphenomena systems. Differential sensitivity is susceptible to errors of the system model and noise of input data and reduces accuracy and stability of the testing procedure. Integral sensitivity tends to reduce zero mean noise, and this effect is amplified when time intervals are properly selected. As seminormalized integral sensitivity of $V_{mj}$ w.r.t $P_i$ on the time interval $[t_a, t_b]$ we understand:

$$W_{ji}[t_a, t_b] = \int_{t_a}^{t_b} S_{ji}(t)\, P_i\, dt, \qquad S_{ij} = \frac{\partial V_{mj}(t)}{\partial P_i}. \quad (5)$$

Choice of the interval $[t_a, t_b]$ influences effectiveness of this technique in reducing noise [2]. We will also use integrated values $U_j[t_a, t_b]$ of the unknowns $V_{mj}(t)$ appearing in the system equation (4):

$$U_j[t_a, t_b] = \int_{t_a}^{t_b} V_j(t)\, dt. \quad (6)$$

### Diagnosis algorithm

The logical steps of the proposed diagnosis algorithm are:

*Step 1. Simulation of nominal system.* Equations (4) describing the system under consideration, are set up in a way discussed in the previous section.

*Step 2. Selection of time intervals for integral sensitivity.* The strategy of selection, aimed at maximal reduction of noise and random errors, is described in detail in [1].

*Step 3. Calculation of integral sensitivity.* Using an integration formula, e.g. trapezoidal, (6) can be replaced by its discretized counterpart:

$$W_{jl}[t_{k-N}, t_k] \approx \approx \sum_{l=0}^{N-1} \frac{S_{jl}(t_{k-l+1}) + S_{jl}(t_{k-l})}{2}(t_{k-l+1} - t_{k-l}), \quad (7)$$

$t_a = t_{k-N}, t_b = t_k$, $N$ - number of discretization points. We apply a similar discretization to $U_j[t_a, t_b]$.

*Step 4. Formulation of diagnosis equations.* Diagnosis equations are obtained by using integral sensitivity, and replacing infinitesimally small changes by finite increments:

$$\mathbf{W}[t_{k-N}, t_k]\ \delta\mathbf{P} = \Delta\mathbf{U}[t_{k-N}, t_k], \quad (8)$$

entries $\delta P_l = \Delta P_l / P_l$ of $\delta\mathbf{P}$ are the relative deviations of parameters from the current vector $\mathbf{P}$; $\Delta U_j[t_{k-N}, t_k]$ is the difference of $U_j$ calculated for the values of variables $V_{mj}(t)$ obtained for the tested and simulated nominal systems. Typically, the number of test equations is much larger than the number of parameters to be identified.

*Step 5. Orthogonalization of the sensitivity matrix.* The orthogonalization technique reduces excessive equations by elimination of linearly or near linearly dependent rows and reduces the number of unknown parameters by detecting ambiguity groups [1].

*Step 6. Solving diagnosis equations and fault detecting.* Diagnosis equations are usually overdetermined. We use this property to provide stability and a good convergence rate of the solving procedure. The regularization technique [5] was designed for solving equations with noisy data. Tikhonov's functional is defined as:

$$\mathbf{M}[\delta\mathbf{P}(\alpha)] = \|\mathbf{W}\delta\mathbf{P}(\alpha) - \Delta\mathbf{U}\|^2 + \alpha\|\delta\mathbf{P}(\alpha)\|^2, \quad (9)$$

$\alpha \geq 0$ is the regularization coefficient. The first term is the standard least square term; the second is added to improve stability and convergence rate of the iterative procedure. Obviously, when $\alpha \to 0$ then $\delta\mathbf{P}$ solves the least square problem. The solution of diagnosis equations is obtained by minimizing the functional:

$$\min_{w.r.t.\,\delta\mathbf{P}} \mathbf{M}[\delta\mathbf{P}(\alpha)] \quad (10)$$

To improve accuracy, (10) is solved iteratively. At the beginning we simulate a fault free system and calculate the required data for the nominal set of parameters $\mathbf{P}^{(1)} = \mathbf{P}^{n}$. A new approximation is calculated as: $\mathbf{P}^{(q+1)} = \mathbf{P}^{(q)}(1 + \delta\mathbf{P}^{(q)})$, and all simulation vectors and matrices are updated. The iterative algorithm stops when: $|\delta P_i^{(q)}| \leq TOL_i \wedge \|\Delta\mathbf{U}\| \leq \varepsilon$. $TOL_i$ - the acceptable tolerance of $\mathbf{P}$, $\varepsilon$ - is the level of noise characteristics of the measurement system. The decision whether the diagnosed parameter $P_i$ is faulty is made by checking the value of $(P_i^n - P_i^{(q)})/P_i^n$ against $TOL_i$. The numerical effectiveness of solving (10) depends on the values of $\alpha$. The optimal value of $\alpha$ for the $q$th iteration is found by solving the equation of generalized discrepancy [5].

The discussed algorithm covers linear systems, but it can be extended to nonlinear systems as well, using an approach similar to [1].

## 4 Test example

We consider the linear fourth order servomechanism [6]. The mechanism (Fig.2) transforms rotational movement of the motor to translational movement of the load. Fig. 3 shows a schematic view of the controller, and Fig.3 the layout of the mechanical part. The following parameters are used: $r(t)$ - reference signal [rad], $T_m$ - servomotor torque [Nm], $C_s$ - servo stiffness [Nm/rad], $D_s$ - servo damping [Nms/rad], $\varphi_1, \dot{\varphi}_1, \ddot{\varphi}_1$ - position, velocity, and acceleration of load [rad, rad/s, rad/$s^2$], $\varphi_2, \dot{\varphi}_2, \ddot{\varphi}_2$ - position, velocity, and acceleration of rotor motor [rad, rad/s, rad/$s^2$], $J_1, J_2$ - inertia load and rotor motor [$kgm^2$], $k$ - stiffness of the transmission [Nm/rad].

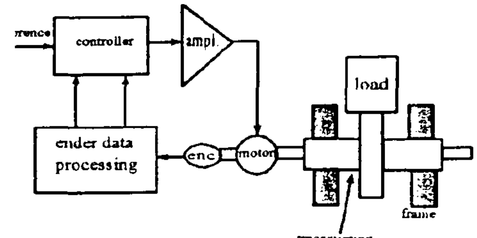
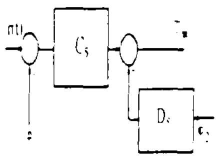



**Fig.2** General layout of the servomechanism



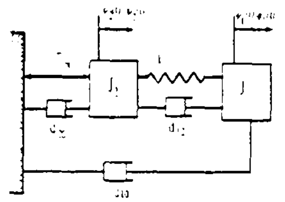


**Fig. 3** Controller **Fig. 4** Mechanical part of servomechanism

The state space equations are [6]:

$$\begin{bmatrix}\dot{\varphi}_1\\ \ddot{\varphi}_1\\ \dot{\varphi}_2\\ \ddot{\varphi}_2\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\dfrac{k}{J_1} & \dfrac{-(d_{10}+d_{12})}{J_1} & \dfrac{k}{J_1} & \dfrac{d_{12}}{J_1}\\ 0 & 0 & 0 & 0\\ \dfrac{k-C_s}{J_2} & \dfrac{d_{12}}{J_2} & -\dfrac{k}{J_2} & \dfrac{-(D_s+d_{12}+d_{20})}{J_2}\end{bmatrix}\begin{bmatrix}\varphi_1\\ \dot{\varphi}_1\\ \varphi_2\\ \dot{\varphi}_2\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ \dfrac{C_s}{J_2}\end{bmatrix}r(t)$$

Diagnosable parameters are: $C_s, D_s, d_{12}, k, J_1, d_{10}$, with the nominal values collected in the vector $\mathbf{P}^n$ (Table 1). The value of $d_{20}$ is fixed at $2.0x10^{-3}\,Nms/rad$ and $J_2$ is $5.0x10^{-4}\ kgm^2$. The testing excitation signal was $\frac{\sin(ax)}{ax}, (a=4)$. A nine order cycloide was used in [6] to examine the tracking accuracy of the servomechanism, but in application to testing this signal leads to sensitivity matrix with a lower column rank. In the identification procedure we detected an ambiguity group $\{C_s, d_{12}\}$. The results of identification are in Table 1: $\mathbf{P}^f, \mathbf{P}^d$ are the faulted and diagnosed vectors of parameters, respectively. As it is shown in the third row of Table 1 the values of faulted parameters $\mathbf{P}^f$ deviate 5% to 30% from their nominal values $\mathbf{P}^n$. The diagnosis procedure has been completed after 17 iterations. The quality of identification (row five) was better then 4%.

## 5 Conclusions

In the paper, the method for fault diagnosis in dynamic multiphenomena (nonlinear) systems has been presented. From the mathematical point of view the discussed method handles, in a uniform manner, equations obtained using various methods of formulation, e.g. nodal modified, state, tableau. The quality of diagnosis is assured by: using time domain integral sensitivity, proper selection of time intervals for sensitivity to reduce numerical and data noise, orthogonalization and regularization techniques to improve accuracy of the solution of overdetermined diagnosis equations. We also examined and showed that the effectiveness of diagnosis depends on the proper selection of testing signals. Using the proposed method the soft faults have been detected in the fourth order servomechanism with a satisfactory accuracy.

## References


[1] V.Brygilevicz, J.Wojciechowski, "Fault diagnosis of dynamic circuit in the presence of measurement noise.". *Proc. ECCTD'95*, Istanbul, (1995), pp.291-294.

[2] V.Brygilewicz, J.Wojciechowski, „Time-domain fault fiagnosis of analogue circuits in the presence of noise," *IEE Proc.- Circuit Devices and Systems*, Vol. 145, 1998, pp.125-131.

[3] A.D.Poularikas and S.Seely, „*Signals and Systems*," PWS Engineering (1985).

[4] V.Brygilevicz, "*Fault diagnosis of dynamic analog systems in the presence of noise*," Ph.D. Thesis, Warsaw University of Technology (1997).

[5]. A.Tikhonov, V.Arsenin, "*Methods for solving incorrect tasks*," Nauka, Moscow 1979.

[6] H.Roelofs and C.Heuvelman, "Optimization of design tolerances of servomechanisms," *Annals of the CIR, vol.41, (1992), pp.467-471*


TABLE 1. Results of diagnosis

| $\mathbf{P}^n$ | $C_s$ 3.423e-1 | $D_s$ 4.607e-2 | $d_{12}$ 4.933e-5 | $k$ 5.99e-1 | $J_1$ 4.40e-4 | $d_{10}$ 5.496e-4 |
|---|---|---|---|---|---|---|
| $\mathbf{P}^f$ | 4.107e-1 | 5.067e-2 | 4.993e-5 | 7.781e-1 | 3.743e-4 | 5.221e-4 |
| $(\mathbf{P}^f-\mathbf{P}^n)/\mathbf{P}^n$ | 20 % | 10 % | 0 % | 30 % | -15% | - 5 % |
| $\mathbf{P}^d$ | 4.213e-1 | 5.138e-2 | 4.933e-5 | 8.145e-1 | 3.668e-4 | 5.012e-4 |
| $(\mathbf{P}^f-\mathbf{P}^d)/\mathbf{P}^f$ | - 2.6% | 1.4 % | 0 % (*) | 4.6 % | - 2 % | - 4 % |