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MULTIFREQUENCY MEASUREMENT OF TESTABILITY IN ANALOG CIRCUITS

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ABSTRACT

An effect of network topology on the testability of linear, dynamic networks is studied. Several methods have been investigated to determine an upper estimate on the rank of the Jacobian matrix associated with network equations. Factorized form of the sensitivity function was obtained to facilitate the symbolic analysis of the Jacobian matrix. An exact, topological method to determine multifrequency measurement of testability is suggested.

I. INTRODUCTION

A recent paper [1] has considered an algorithm to evaluate the measure of testability in linear, analog systems. The measure is based on the rank of the Jacobian matrix of the response vector determined for different frequencies. Consider a linear system described by the nodal equations

$$\underset{\sim}{Y}(\underset{\sim}{p},s)\begin{bmatrix}\underset{\sim}{y}(\underset{\sim}{p},s)\\ \underset{\sim}{z}(\underset{\sim}{p},s)\end{bmatrix}=\begin{bmatrix}\underset{\sim}{x}(s)\\ \underset{\sim}{0}\end{bmatrix}, \qquad (1)$$

where $\underset{\sim}{p}=[p_k]$, with $k=1,\dots,\ell$. Using results from [2] Iuculano et al. evaluate the maximum rank R_{max} of the Jacobian matrix $\underset{\sim}{\Phi}$ of the system response to different exciations

$$\underset{\sim}{y}(\underset{\sim}{p},s)=\underset{\sim}{H}(\underset{\sim}{p},s)\,\underset{\sim}{x}(s) \qquad (2)$$

calculated for any possible set of test frequencies. The response and excitation vectors $\underset{\sim}{y}(\underset{\sim}{p},s)$ and $\underset{\sim}{x}(s)$ have m and e elements respectively.

Since elements h_{ij} of the transfer function matrix $\underset{\sim}{H}(\underset{\sim}{p},s)$ are rational functions of the complex frequency s, the rank of $\underset{\sim}{\Phi}$ can be related to the degree of polynomials of s in the numerator and the denominator of each function. Let (i,j)th element of $\underset{\sim}{H}(\underset{\sim}{p},s)$ be represented in the following form:

$$h_{ij}=\frac{\Delta_{ji}(\underset{\sim}{p},s)}{\Delta(\underset{\sim}{p},s)}, \quad \begin{matrix}i=1,\dots,m,\\ j=1,\dots,e,\end{matrix} \qquad (3)$$

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where $\Delta(\underset{\sim}{p},s)$ and $\Delta_{ji}(\underset{\sim}{p},s)$ are the determinant of $\underset{\sim}{Y}(\underset{\sim}{p},s)$ and the first order cofactor of its (j,i)th element, respectively. The Jacobian matrix

$$\underset{\sim}{\Phi}=[\Phi_{rk}], \qquad \begin{matrix}r=1,\dots,e\cdot m,\\ k=1,\dots,\ell,\end{matrix} \qquad (4)$$

with

$$\Phi_{rk}=\frac{\partial h_{ij}}{\partial p_k}, \quad \text{where } r=(j-1)m+i,$$

can be obtained from (3) by differentiation

$$\Phi_{rk}(\underset{\sim}{p},s)=\frac{1}{\Delta^2(\underset{\sim}{p},s)}\left[\frac{\partial\Delta_{ji}(\underset{\sim}{p},s)}{\partial p_k}\Delta(\underset{\sim}{p},s)-\frac{\partial\Delta(\underset{\sim}{p},s)}{\partial p_k}\Delta_{ji}(\underset{\sim}{p},s)\right]$$

$$\overset{\Delta}{=}\frac{1}{\Delta^2(\underset{\sim}{p},s)}P_{rk}(\underset{\sim}{p},s), \qquad (5)$$

where $P_{rk}(\underset{\sim}{p},s)$ are real polynomials of s.

Using Taylor's expansion of vector obtained from $\underset{\sim}{H}(\underset{\sim}{p},s)$ around the nominal point $\underset{\sim}{p}_0$ we obtain

$$\text{vec}[\underset{\sim}{H}(\underset{\sim}{p},s)]\simeq\text{vec}[\underset{\sim}{H}(\underset{\sim}{p}_0,s)]+\underset{\sim}{\Phi}(\underset{\sim}{p}_0,s)\,\Delta\underset{\sim}{p}, \qquad (6)$$

and the rank R of $\underset{\sim}{\Phi}(\underset{\sim}{p}_0,s)$ determines how many elements of $\Delta\underset{\sim}{p}$ can be uniquely evaluated. Obviously $R\le\ell$ and for a single frequency measurements $R\le e\cdot m$. Measuring the system responses to the excitations at different frequencies, R can be increased beyond $e\cdot m$, therefore more elements can be evaluated.

In [1] the authors have shown that the maximum rank of $\underset{\sim}{\Phi}$, calculated over any possible set of test frequencies R_{max} is equal to the rank of the real matrix $\underset{\sim}{B}_{t\times\ell}$, whose kth column is obtained from the coefficients of the polynomials $P_{rk}(s)$, $r=1,\dots,e\cdot m$. An upper estimate of rows t of this matrix adopted in [1] is based on the number of reactive elements n_0

$$t=(2n_0+1)\cdot e\cdot m \qquad (7)$$

Consequently the authors suggest that if all the circuit elements are frequency dependent ($\ell=n_0$) the number of rows of $\underset{\sim}{B}$ becomes equal to $(2\ell+1)\cdot e\cdot m$ and maximum testability could be reached also with small values of the the product $e\cdot m$. According to our judgement this is an oversimplification of what appears to be a complex problem, and the true values of R_{max} may be well below the estimate (7).

In this paper we address the problem of an effect of the network topology on the rank of the Jacobian matrix obtained with multifrequency measurements. In what follows a relationship between R_{max} and the order of complexity of an electrical network is discussed. Next, we derive a factorized form for $\underset{\sim}{P}_{rk}(\underset{\sim}{p},s)$ and link it to the network transfer functions. Rank limitations which come from the network decomposition is discussed in Section IV and finally a method based on the fully symbolic analysis is presented.

II. TOPOLOGICAL CONSTRAINTS.

Presented in [1] the idea of evaluation of R_{max} on the basis of the real matrix is extremely useful in the multifrequency fault diagnosis. It allows us to determine the number of test points necessary to evaluate network elements and, as such, is helpful in designing for testability and evaluation of the testability measure for a given set of test points.

To link the network topology to the degrees of polynomials $\underset{\sim}{P}_{rk}(\underset{\sim}{p},s)$ recall the order of complexity of electrical network N [3], defined as the number of nonzero zeros of the determinant Δ. From [3] we know that

$$N = \rho_{C_o} + \rho_{L_o} + \rho_{C_s} + \rho_{L_s} , \qquad (8)$$

where ρ_{Co} (ρ_{Lo}) is the rank of the graph obtained from the network after open-circuiting all elements which are not capacitors, (inductors) respectively, ρ_{Cs} (ρ_{Ls}) is the rank of the graph obtained from the network after short-circuiting all elements which are not capacitors, (inductors) respectively.

Note that the order of complexity N is always less or equal to the number of reactive elements. Based on the evaluated order of complexity we can justify the following corollary.

<u>Corollary</u>

An upper bound on the degrees of the polynomials $P_{rk}(\underset{\sim}{p},s)$ is equal to 2N. Consequently the upper bound on R_{max} is $t_o' = (2N + 1)\cdot e\cdot m$.

Even tighter estimate of R_{max} comes from adoption of results presented in [4], where the network solvability was linked for the first time to the degrees of polynomials of numerators and denominator of measured transfer functions. If we estimate the number of nonzero coefficients in each such polynomial by N+1, then according to [4] the number of elements one can evalute on the basis of given measurements (i.e. R_{max}) is limited by

$$t_o = N\cdot e\cdot m + e\cdot m + N \quad ,$$

which is less than t_o'. This results mean that not all reactive elements in the network contribute to the rank of the Jacobian matrix (4), and that the topology of the network is crucial in establishing its upper bound.

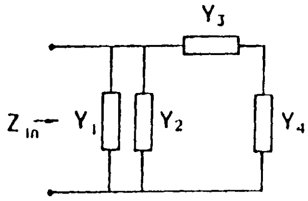

Fig. 1. An example of one-port network.

Another observation one can make is that a relative location of the reactive elements and the measurement points is equally important. The same network with the same number of test points may have different rank of $\underset{\sim}{\Phi}$ depending on the location of the test points.

<u>Example</u>

Consider a one-port network shown in Fig. 1. Its driving point impedance Z_{in} is equal to

$$Z_{in} = \frac{\Delta_{11}}{\Delta} = \frac{Y_3 + Y_4}{(Y_1 + Y_2)(Y_3 + Y_4) + Y_3Y_4} ,$$

where Y_i is the admittance of ith element. In this example we have e=m=1. The Jacobian matrix of Z_{in} is expressed by

$$\underset{\sim}{\Phi}(s) = \left.\frac{\partial Z_{in}}{\partial p_k}\right|_{k=1,\dots,4} = \left.\frac{\partial Z_{in}}{\partial Y_k}\frac{\partial Y_k}{\partial p_k}\right|_{k=1,\dots,4}$$

$$= \frac{1}{\Delta^2}\left[-(Y_3+Y_4)^2\frac{\partial Y_1}{\partial p_1},\ -(Y_3+Y_4)^2\frac{\partial Y_2}{\partial p_2},\ -Y_4^2\frac{\partial Y_3}{\partial p_3},\ -Y_3^2\frac{\partial Y_4}{\partial p_4}\right] .$$

Consider three cases.

I. All elements are capacitors with $Y_i = sC_i$. In this case $n_o=4$, $\rho_{Co}=\rho_{Cs}=2$, $\rho_{Lo}=\rho_{Ls}=0$, N=0, $t_o=1$, and

$$\underset{\sim}{\Phi}(s) = \frac{s^3}{\Delta^2}\,[\ -(C_3+C_4)^2,\ -(C_3+C_4)^2,\ -C_4^2,\ -C_3^2\ ] .$$

Obviously $R_{max}=1$, while an estimate based on (7) is t=9. This case well illustrates the effect of the order of complexity on the rank of $\underset{\sim}{\Phi}$.

II. Let $Y_4=sC_4$, and $Y_i=G_i$, i=1,2,3. In this case $n_o=1$, t=3, $\rho_{Co}=1$, $\rho_{Cs}=\rho_{Lo}=\rho_{Ls}=0$, N=1, $t_o=3$, and

$$\underset{\sim}{\Phi}(s) = \frac{1}{\Delta^2}\,[-(G_3+sC_4)^2,\ -(G_3+sC_4)^2,\ -s^2C_4^2,\ -sG_3^2] ,$$

with $R_{max}=3$. Here we have an agreement between predicted and estimated value of the rank.

III. Let $Y_1=sC_1$, and $Y_i=G_i$, i=2,3,4. In this case $n_o=1$, t=3, $\rho_{Co}=1$, $\rho_{Cs}=\rho_{Lo}=\rho_{Ls}=0$, N=1, $t_o=3$, and

$$\underset{\sim}{\Phi}(s) = \frac{1}{\Delta^2}\,[\ -s(G_3+G_4)^2,\ -(G_3+G_4)^2,\ -G_4^2,\ -G_3^2\ ] ,$$

with R_{max}=2. The predicted rank is overestimated as the effect of the relative location of the reactive element w.r.t. the measurement.

III. FACTORIZED SENSITIVITY

In this section polynominals $P_{rk}(\underset{\sim}{p},s)$ of (5) are expressed in terms of the products of the cofactors of the coefficient matrix. On such basis the evaluation of the rank R should be easier and directly linked to the transfer functions of the analyzed network.

Consider the transfer function h_{ij} described by (3). Its derivative w.r.t. the parameter p_k can be expressed as

$$\frac{\partial h_{ij}}{\partial p_k} = \frac{\partial h_{ij}}{\partial Y_k}\frac{\partial Y_k}{\partial p_k} \quad , \tag{9}$$

where Y_k is the admittance of p_k, and

$$\frac{\partial h_{ij}}{\partial Y_k} = \frac{\dot{\Delta}_{ji}\Delta - \dot{\Delta}\,\Delta_{ji}}{\Delta^2} \quad , \tag{10}$$

where $\dot{\Delta}_{ji}$, $\dot{\Delta}$ denote the derivatives of the corresponding cofactors w.r.t. Y_k. Let the admittance Y_k be located in the coefficient matrix $\underset{\sim}{Y}$ on the intersection of the rows a, b and the columns c, d as follows

$$\begin{array}{c} \\ a \\ b \end{array}\begin{array}{c} \begin{array}{ccccc} & c & & d & \end{array} \\ \left[\begin{array}{ccccc} .. & Y_k & ... & -Y_k & \\ .. & -Y_k & ... & Y_k & \end{array}\right] \end{array} \tag{11}$$

It can be shown that

$$\begin{aligned} \Delta\,\dot{\Delta}_{ji} &= \Delta\,\Delta_{ac,ji} - \Delta\,\Delta_{ad,ji} - \Delta\,\Delta_{bc,ji} + \Delta\,\Delta_{bd,ji} \\ &= \Delta_{ac}\Delta_{ji} - \Delta_{ai}\Delta_{jc} - \Delta_{ad}\Delta_{ji} + \Delta_{ai}\Delta_{jd} \\ &\quad - \Delta_{bc}\Delta_{ji} + \Delta_{bi}\Delta_{jc} + \Delta_{bd}\Delta_{ji} - \Delta_{bi}\Delta_{jd} \ . \end{aligned} \tag{12}$$

On the other hand

$$\dot{\Delta}\,\Delta_{ji} = \Delta_{ac}\Delta_{ji} - \Delta_{ad}\Delta_{ji} - \Delta_{bc}\Delta_{ji} + \Delta_{bd}\Delta_{ji} \ , \tag{13}$$

so

$$\dot{\Delta}_{ji}\Delta - \dot{\Delta}\,\Delta_{ji} = -(\Delta_{jc} - \Delta_{jd})(\Delta_{ai} - \Delta_{bi}) \ . \tag{14}$$

From (10) we obtain

$$\frac{\partial h_{ij}}{\partial Y_k} = \frac{-(\Delta_{jc} - \Delta_{jd})(\Delta_{ai} - \Delta_{bi})}{\Delta^2} \ . \tag{15}$$

Since $h_{ij}=\Delta_{ji}/\Delta$ is a transfer function, which determines the response y_i due to the excitation x_j we can write

$$\frac{\partial h_{ij}}{\partial Y_k} = - h_{(c-d)j}h_{i(a-b)} \ , \tag{16}$$

where the transfer function $h_{(c-d)j}$ determines the response (y_c-y_d) measured between the nodes c and d due to the excitation x_j, and $h_{i(a-b)}$ determines the response y_i due to the excitation applied between the nodes a and b.

In the particular case, when $\underset{\sim}{T}$ is the indefinite admittance matrix of a network, then (15) can be reduced to

$$\frac{\partial h_{ij}}{\partial Y_k} = - \frac{T_{jc,rd}\,T_{ai,br}}{T^2_{uv}} \quad , \tag{17}$$

where T_{uv}, $T_{jc,rd}$, $T_{ai,br}$ are the first and the second order cofactors of $\underset{\sim}{T}$. Using (17) one can determine the degree of polynomials in $P_{rk}(\underset{\sim}{p},s)$.

IV. MULTITERMINAL NETWORK LIMITATION

As could be seen from the case III of Example, the order of complexity alone is not the only topological information one needs to evaluate the rank of $\underset{\sim}{\Phi}$. To derive more accurate estimate of R_{max} the effect of the reactive elements on the input-output relationships must be considered. Assume for simplicity that the network contains capacitors as the only frequency-dependent elements, and that the excitation and measurement nodes are the same (m=e).

If all reactive elements are incident to the measurement nodes only, then the coefficient matrix $\underset{\sim}{Y}(\underset{\sim}{p},s)$ can be represented as

$$\underset{\sim}{Y}(\underset{\sim}{p},s) = \begin{bmatrix} G_{11}(\underset{\sim}{p})+sC_{11}(\underset{\sim}{p}) & G_{12}(\underset{\sim}{p}) \\ G_{21}(\underset{\sim}{p}) & G_{22}(\underset{\sim}{p}) \end{bmatrix} \ , \tag{18}$$

where the subscripts 1 and 2 correspond to the measurement nodes and all the remaining nodes respectively. In such a case

$$\underset{\sim}{H}^{-1}(\underset{\sim}{p},s) = G_{11}(\underset{\sim}{p})+sC_{11}(\underset{\sim}{p})-G_{12}(\underset{\sim}{p})G^{-1}_{22}(\underset{\sim}{p})G_{21}(\underset{\sim}{p}) \ . \tag{19}$$

Changing excitations and measuring responses we can identify $\underset{\sim}{H}(\underset{\sim}{p},s)$, therefore from (19) we can find

$$sC_{11}(\underset{\sim}{p}) = \underset{\sim}{H}^{-1}(\underset{\sim}{p},s) - \underset{\sim}{H}^{-1}(\underset{\sim}{p},0) \ , \tag{20}$$

and all the frequency dependent elements can be directly identified when the measurements are taken for DC excitations and then repeated for single, nonzero frequency. This result also indicates that adding reactive elements between the network terminals cannot increase the rank R_{max} by more than the number of reactive elements added.

It is well known [4], that in the resistive network $R_{max} \le m^2$. In the case of symmetrical resistive network $R_{max} \le m(m+1)/2$. Based on the above observation we have the following result.

Result 1

If all the reactive elements are incident to the measurement nodes only then the upper bound on R_{max} is $t_1 = m^2 + n_0$.

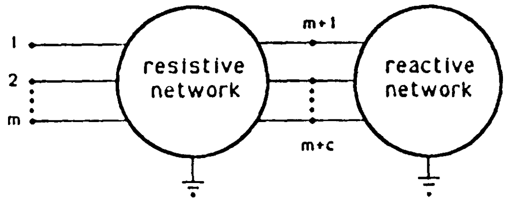

Fig. 2. Resistive and reactive subnetworks.

This result can be used to explain why $R_{max}=2$ in the case III of Example. Another obvious limitation comes from the network decomposition.

Result 2

Consider two subnetworks: N_1 with the rank of its Jacobian matrix R_1 and N_2 with the rank R_2. If we combine these two subnetworks preserving their excitation and measurement points, then the rank R_{max} for the combined network is less or equal to $R_1 + R_2$.

If the number of reactive elements is small, comparing to the number of nodes, we can use results 1 and 2 to obtain better estimate for the rank R_{max}. Let us represent coefficient matrix of such a network in the form (18) assuming that the subscript 1 corresponds to all the accessible nodes as well as those incident to the reactive elements. Let m+c denotes cardinality of this set of nodes. This decomposition is illustrated on Fig. 2, in which we have assumed that the reactive elements are incident to the internal nodes only. Even if all m+c terminals shown in Fig. 2 were accessible, the rank R_{max} for the combined network is less or equal to $R_1 + R_2$, where $R_1 \leq (m+c)^2$ and $R_2 \leq n_0$ so we can formulate the following result.

Result 3

An upper bound on R_{max} in the frequency dependent linear network is $t_2 = (m+c)^2 + n_0$.

Partition shown in Fig. 2 can be used to separate effect of reactive and resistive elements. Using excitations and responses on terminals m+1 to m+c at a single nonzero frequency one can identify no more than c^2 reactive elements. If the network is symmetrical then we can obtain a special case of Result 3 formulated as follows.

Result 4

An upper bound on R_{max} in the frequency dependent linear, symmetrical network is

$$t_3=[(m+c)(m+c+1)+c^2+c]/2 \; .$$

Finally if the excitation and measurement nodes are different, then t_2 in Result 3 should be replaced by

$$t_4= (m+c)(e+c)+R_2 .$$

V. RANK TEST BASED ON TOPOLOGICAL ANALYSIS

In [5] the elements of a resistive network were identified using topological analysis tools. A topological matrix $\underset{\sim}{B}$, which represents 2-trees needed to generate numerators of transfer functions measured, was used. The rank of the Jacobian matrix $\underset{\sim}{\Phi}$ have been shown to be a function of the rank of a selected submatrix of this topological matrix. The submatrix must contain exactly one 2-tree from each transfer function.

To our knowledge this is the most exact method of rank estimation based on the network topology, but it is also the most expensive one since it requires fully symbolic analysis of a given network. This method can be applied to find rank R_1 and R_2 in the separated subnetworks and estimate R_{max} on the basis of the Result 2.

One can think of a new method applied directly to the network which contains both resistive and reactive elements. We have tested a method which modifies approach presented in [5] distinguishing 2-trees of different degree of complex variable s. The submatrix of the topological matrix $\underset{\sim}{B}$ contains selected 2-trees. Each 2-tree has a distinct degree of s for each transfer function. Results of our research on this new method are to be reported.

CONCULSIONS

The rank of the Jacobian matrix of the network equations in multifrequency measurements depends on the order of complexity of the analyzed network as well as on the relative location of reactive elements and the test points. Decomposition of the network can be used to separate an effect of reactive and resistive elements. In the decomposed network we can analyze each part for testability to obtain an upper estimate on the rank of the Jacobian matrix. An exact method of rank estimation requires fully symbolic analysis of a given network. This method, although the most expensive, is very accurate and can be used to develop an efficient testing strategy in the large, dynamic networks.

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