# Fault Diagnosis of Nonlinear Resistive Circuits

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## ABSTRACT

This paper addresses the problem of fault diagnosis in nonlinear resistive circuits. Fault diagnosis equations are formulated with the nonlinear resistors being approximated by piecewise linear segments. By varying excitations, the non-linearity of the circuit is exploited to gain more independent measurements using the same test nodes. These additional measurements are used to formulate linear fault diagnosis equations that can be solved to locate the faulty elements.

## I. INTRODUCTION

Fault diagnosis of nonlinear circuits has become an increasingly important issue in circuit design, fabrication and maintenance [1–2]. Different methods have been proposed for nonlinear testing. The linear programming technique was used in [3], where the nonlinear elements were replaced by their linearized model. In [4] the network was decomposed into linear and nonlinear parts, and under the assumption that all nonlinear elements are accessible necessary, and sufficient conditions for the testability of the nonlinear network were derived.

In this paper we present a method for multiple fault location of nonlinear resistive circuits based on the fault verification principle [5–6]. The nonlinear resistors are approximated by piecewise linear segments. Consequently at the $\ell$th segment the equation representing any nonlinear voltage controlled resistor

$$i_\ell = i_\ell^0 + g_\ell v \tag{1}$$

where $g_\ell$ is the slope of the $\ell$th segment and $i_\ell^0$ is the cut–off current.

The system of equations describing the circuit can, therefore, be written as

$$T_\ell X_\ell = I + I_\ell \tag{2}$$

where $T_\ell$ is the coefficient matrix representing the circuit, $X_\ell$ is the output vector, $I_\ell$ represents cut–off currents and $I$ represents the independent current sources with $\ell$ denoting the region in which the circuit operates. It should be noted here that in every region a nonlinear resistor is identified through two parameters, the slope and the cut–off current, rather than the slope only, as in the case of linear resistors.

The basic procedure we follow in this paper is the so called assume–and–check method. The faults are assumed to be within an f element set and the remaining elements are verified to be fault–free. This verification is carried out by checking the consistency of a set of diagnosis equations which are invariant on faulty elements. If the assumption is proved to be wrong, another set of f elements are chosen and the verification procedure is updated.

This paper is organized as follows. In Section II the fault diagnosis equations are formulated. The solution of these diagnosis equations is discussed in Section III with an illustrative example. A verification procedure with the classification of faults into linear faults for linear elements and nonlinear faults for nonlinear elements is presented in Section IV with another example.

## II. FAULT DIAGNOSIS EQUATIONS

Assume that f–elements out of b of the circuit N are faulty with the other (b–f) elements fault–free. Accordingly, we denote $\phi_f$ as the vector of faulty elements which is given by

$$\phi_f = [y_1, y_2, \ldots, y_f]^T . \tag{3}$$

But since any circuit output function $x_j$ can be represented in a symbolic form as a ratio of two polynomials in $\phi_f$ [5], $x_j$ can be represented in the following form

$$x_j = \frac{P_j}{Q_j} . \tag{4}$$

Therefore

$$x_j Q_j - P_j = 0 . \tag{5}$$

Substituting for $P_j$ and $Q_j$, we get

$$T_0^j + \sum_{i=1}^{f} T_i^j y_i + \sum_{i_1=1}^{f} \sum_{i_2=1}^{f} T_{i_1 i_2}^j y_{i_1} y_{i_2} + \ldots + T_{i_1 \ldots i_f}^j y_1 y_2 \cdots y_f = 0 \tag{6}$$

where the values of the coefficients T depend only on the values of the fault–free elements. For different outputs, equations similar to (6) are formulated. This set of equations is called fault diagnosis equations. A significant fact here is that the nonlinearity of the circuit can be exploited to increase the number of independent measurements by varying the excitation currents. In fact, if the excitation current is changed slightly, in such a way that the region of operation is not altered, we will have another measurement $x_j$ that can be used in the fault location process. In other words, the same output with different excitations, more equations of the form (6) can be added to the set of diagnosis equations. The number of independent measurements that we can get depends on the topology of the network and the number of piecewise linear segments used to approximate the characteristics of the nonlinear resistors.

## III. SOLUTION OF THE DIAGNOSIS EQUATIONS

We need (f+1) equations as (6) to verify the fault assumption, since f of these nonlinear equations can be solved using any iterative method. The values of the elements obtained are substituted in the remaining equation. If this equation is satisfied, the assumption is correct; otherwise a different set of faulty elements is chosen. However, due to the structure of the diagnosis equations, we can form a set of linear diagnosis equations that can be easily solved to find the faulty elements. This linearization procedure [5–6] can be explained in the following way.

Since the number of nonlinear terms in (6) is no more than $(2^f - f - 1)$, therefore having $2^f - 1$ independent equations as

(6), we can eliminate the nonlinear terms. This, in turn, will lead to a system of f–linear equations in f unknowns. The solution is used to check whether the remaining equations are satisfied. It is clear that more independent measurements will be needed but the advantage is that we will end up solving a system of linear equations. It is also worth mentioning that different circuit functions should be used to formulate the diagnosis equations rather than only one functions as in [5] which might be insensitive to some elements.

Example 1

Consider the circuit of Fig. 1 with two nonlinear resistors $R_3$ and $R_5$. The piecewise linear segments that approximate the characteristics of $R_3$ and $R_5$ are shown in Fig. 2. Assume that the nominal values of the linear resistors are $R_i = 1\Omega$, i = 1, 2, 4 and the outputs are the nodal voltages $V_1$, $V_2$ and $V_3$.

The circuit nodal equations are given by

$$\begin{bmatrix} g_1+g_2 & -g_2 & 0 \\ -g_2 & g_2+g_3+g_4 & -g_4 \\ 0 & -g_4 & g_4+g_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_s \\ I_3 \\ I_5 \end{bmatrix} \quad (7)$$

We simulate the following faults. $R_2$ has increased from 1 to 2 and the characteristic of $R_5$ has changed as shown in Fig. 3. With $I_s = 6$, the measured output voltages are $V_1 = 4.45$, $V_2 = 1.35$ and $V_3 = 0.65$ V. It should be noted that these voltages may as well be computed using the algorithm given in [7]. Using (7) and substituting the nominal values of the assumed fault–free elements $g_1$, $g_3$, $g_4$, we can represent the nodal voltages in terms of $g_2$, $g_5$ and $I_5$. Utilizing the measured values of $V_1$, $V_2$ and $V_3$, we get the following diagnosis equations:

$$1.8\, g_2 - 3.1\, g_5 + 6.85\, g_2 g_5 = 1.55 \quad (8)$$

$$2.2\, g_2 - 0.8\, g_5 - I_5 + 3.55\, g_2 g_5 - g_2 I_5 = 2.15 \quad (9)$$

$$5.2\, g_2 - 1.3\, g_5 + 2I_5 - 1.95\, g_2 g_5 + 3g_2 I_5 = 0.15 \quad (10)$$

Equations (8), (9) and (10) can be solved iteratively to find $g_2$, $g_5$ and $I_5$. We can also use the linearization procedure described in the previous section to formulate linear fault diagnosis equations. This can be done by changing the excitation current slightly in such a way that we stay within the same linear segments of the nonlinear resistors. Another set of three diagnosis equations can be formulated. Consequently, the nonlinear terms $g_2 g_5$ and $g_2 I_5$ are eliminated and we obtain three linear equations in three unknowns $g_2$, $g_5$ and $I_5$.

## IV. FAULT VERIFICATION WITH SEPARATION OF FAULTS

With the classification of faults as linear faults for faulty linear resistances and nonlinear faults for faulty nonlinear resistances, we propose a fault verification technique that exploit the nonlinearity of the circuit to first identify linear faults and then using this information to identify nonlinear faults. This can be achieved by driving the circuit using different linear segments of the piecewise linear resistors. The procedure is illustrated in the following example.

Example 2

Consider again the circuit of Fig. 1 with the same set of faults as before. Therefore, there is a linear fault $R_2$ and nonlinear fault $R_5$. With $I_s = 2$, the measured output voltages are $V_1 = 1.545$, $V_2 = 0.636$ and $V_3 = 0.318$ V. Assuming that $g_2$ is the only fault at this operating point, we get the following diagnosis equations

$$1.545\,(3+5g_2) - (5g_2+6) = 0 \quad (11)$$

$$0.318\,(3+5g_2) - (0.5+2.5g_2) = 0 \quad (12)$$

from which we find that $g_2 = 0.5$ and the assumption that $g_2$ is faulty is verified since (12) is satisfied.

Now using this value of $g_2$ and with $I_s = 6.0$, the values of the measured output voltages will be as given in Example 1. We get the following diagnosis equations

$$3.1\, g_5 = 6.2 \quad (13)$$

$$0.975\, g_5 - 1.5\, I_5 = 1.05 \quad (14)$$

from which we can find $g_5$ and $I_5$.

## CONCLUSIONS

The problem of fault diagnosis of nonlinear resistive circuits is investigated. Fault diagnosis equations are formulated assuming that the faults are within f element set and the remaining elements are fault–free. In these equations which are invariant on faulty elements, the characteristics of the nonlinear resistors are approximated by piecewise linear segments. By varying the excitations, the nonlinearity of the circuit is exploited to obtain additional independent measure–ments. These measurements can be used to linearize the diagnosis equations. If the solution of these linear equations satisfies the remaining fault diagnosis equations, then the faults are located. Otherwise, a different set of faults is assumed and the procedure is repeated. A special procedure that separates between linear and nonlinear fault is also suggested. Illustrative examples are also given.

## REFERENCES


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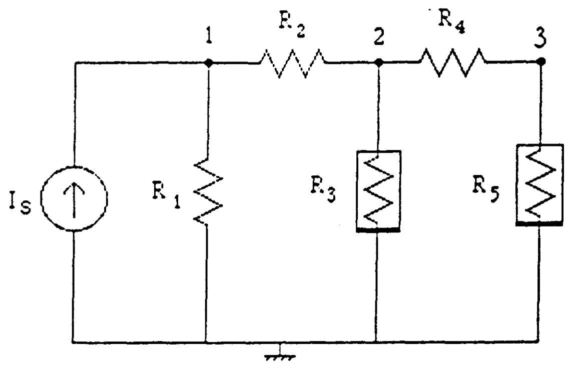


Fig. 1 Nonlinear resistive circuit.

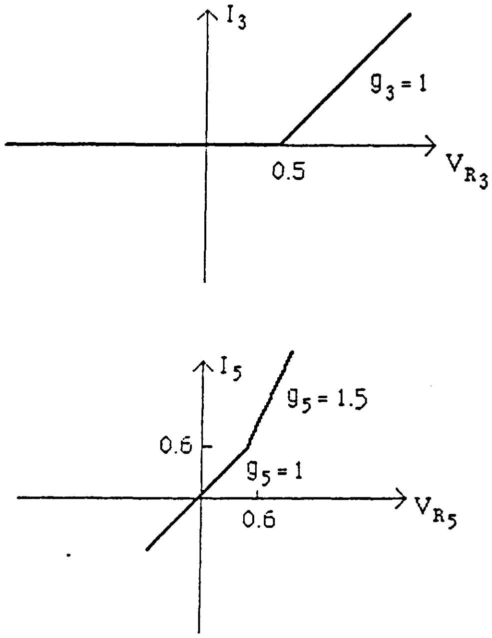


Fig. 2 Piecewise linear characteristics of $R_3$ and $R_5$.

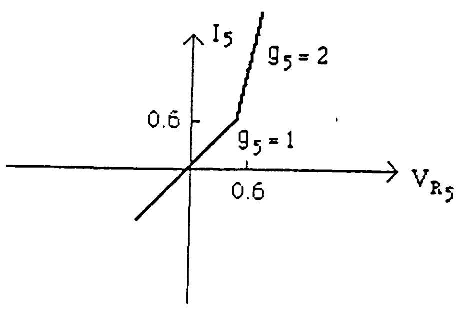


Fig. 3 Piecewise linear characteristics of the faulty resistor $R_5$.