ELEMENT EVALUATION IN THE RESISTIVE NETWORKS

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Abstract

This paper presents an efficient technique for the element evaluation in a large grid resistive network. The method is based on the optimization techniques applied in so called layer by layer fashion. Computational requirements of this method are compared with those of the direct optimization technique used in the impedance computerized tomography. An example is given to illustrate the method presented.

I. INTRODUCTION

The need for the element evaluation technique in a large scale networks stems from the requirements of the impedance computerized tomography (ICT) technique used in medical diagnosis, subsurface mineral exploration and material testing. The ICT technique is being developed since 1978. In contrast to the conventional tomography techniques, this technique does not use x-rays, but rather employs a weak electrical current to map out the electrical properties of the tissues in a tomographic slice. It is expected to have the following advantages: (1) reduced biological hazard for medical application, (2) less expensive hardware, (3) expansibility to full 3-D image, and (4) very high data rates.

In the ICT line integrals along curved paths through the object must be solved. These paths are object dependent, and hence their coordinates are not known in advance. As a result, the reconstruction process involves the solution of a nonlinear system of equation. Several methods have been developed to solve this nonlinear problem. One is the guarding technique suggested by Price [1]. This technique helps to minimize the nonlinearity of current paths. But unfortunately, it does not force the current to flow straight in an inhomogeneous medium, no matter how good the guard electrodes are. In 1978 M. Tasto developed the method [2], in which the resistivity profile of an object is corrected by backprojecting the difference between the measured and the calculated resistance integral along the current path. M. Tasto's method can only be used in the case of small variations of resistivity. Similar to Tasto's approach is the sensitivity method [3], in which the resistivity profile of an object is corrected by backprojecting the difference between the measured and calculated current density.

Network approach to the ICT was developed by K. A. Dines and R. J. Lytle [4]. In their method, a network model serves as a discrete approximation of the distributed - parameter system. Its conductivity image is generated by runing an iterative process on network equations that are linearized in unknown conductance variables. Computer simulation studies using data generated from the network model show good results and demonstrate feasibility of this approach. The main limitation for this method evolves from a large number of equations one must solve simultaneously, which take a huge computer memory and require an exorbitant execution time. Also the error for elements evaluated in the center is higher than for those at the boundary.

In this paper a method which solves the nonlinear problem through the network decomposition layer by layer is introduced. Section II presents the concept of the element evaluation based on the direct method. Section III discribes the layer by layer method and investigates computational requirements of the proposed algrithm. Section IV includes an example to demonstrate capabilities of the method.

II. DIRECT METHOD

The element evaluation problem is to determine values of the elements uniquely from the network's behavior as seen from its external nodes. For a given network, the transfer admittance is a function of the element values and the topology of the network.

$$y_{k\ell} = y_{k\ell}\,(\,x_1,\ x_2,\ \ldots\,) \qquad (1)$$

where

$$y_{k\ell} = \left.\frac{i_k}{v_\ell}\right|_{v_x = 0,\ x \neq \ell}$$

Assume the network has N elements and (m+1) external nodes. The number of independent measurements taken from external nodes is M = m(m+1)/2. Formulate the system equations as [5]

$$\underset{\sim}{F}(\underset{\sim}{X}) = \underset{\sim}{0} \qquad (2)$$

with $\underset{\sim}{X} = [\,x_1,\ x_2,\ \ldots,\ x_N\,]^T$

and $\underset{\sim}{F} = [\,f_1,\ f_2,\ \ldots,\ f_M\,]^T$

These equations are nonlinear and can be solved iteratively using Newton-Raphson algorithm

$$\underset{\sim}{J}^k \Delta\underset{\sim}{X}^k = - \underset{\sim}{F}^k \tag{3}$$

$$\underset{\sim}{X}^{k+1} = \underset{\sim}{X}^k + t^k \Delta\underset{\sim}{X}^k \tag{4}$$

where $\underset{\sim}{J} = \dfrac{\partial \underset{\sim}{F}(\underset{\sim}{X})}{\partial \underset{\sim}{X}}$ is the Jacobian matrix, $0 < t < 1$ and k indicates the kth iterative step.

A necessary and sufficient condition for the local solvability is that the system (3) is overdetermined and consistent, i.e.,

$$M > N \tag{5}$$

and $$\text{rank}(\underset{\sim}{J}) = \text{rank}(\underset{\sim}{J}') = N \tag{6}$$

where $\underset{\sim}{J}'$ is the augmented Jacobian matrix [5].

The grid resistive network is chosen as a model (Fig. 1) and each edge of a grid represents an unknown admittance. In this model we define a layer as a subnetwork, which contains elements adjacent to the perimeter of the current network. If the grid network has L layers, then the number of edges is

$$N = 8L^2 + 4L \tag{7}$$

and the number of independent measurements is

$$M = m(m+1)/2 = 32L^2 - 4L \tag{8}$$

It is obvious that $M > N$ for $L > 1$ and computer results shows that the system is consistent for any size of grid models. So the grid model satisfies the condition of local solvability. The elements values can be determined provided that good initial guess is given.

The Jacobian of the system (3) is a M x N matrix and its size increases largely with the size of the model. Refering to (7) and (8) we can approximate the area required by $256L^4$. In medical applications the minimum size of the model should be 100 layers to obtain a clear picture. In this case, the system will have 2.56×10^{10} coefficients.

In order to reduce the computational effort, the elements can be evaluated using multiple steps procedure instead of using the direct evaluation of all elements, as discussed in the next section.

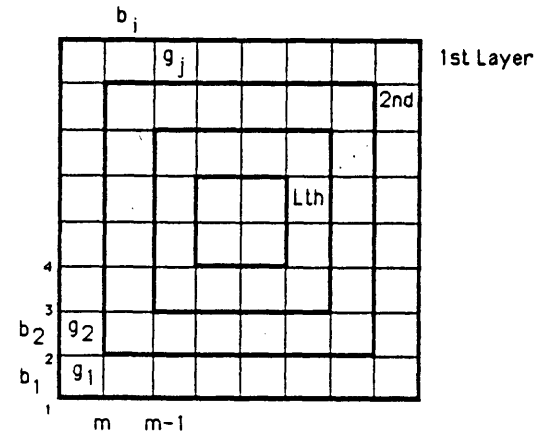


Fig. 1 The grid resistive network

III. LAYER BY LAYER METHOD

In what follows we describe a decomposition approach developed for the purpose of the elements evaluation in a resistive grid network.

Divide the grid model into L layers. Lable layers staring from the boundary to the center. The node-admittance matrix $\underset{\sim}{Y}_{nn}$ is a band matrix as follows:

$$\underset{\sim}{Y}_{nn} = \begin{bmatrix} \underset{\sim}{Y}_{11} & \underset{\sim}{Y}_{12} & & & & \\ \underset{\sim}{Y}_{21} & \underset{\sim}{Y}_{22} & \underset{\sim}{Y}_{23} & & \underset{\sim}{0} & \\ & \underset{\sim}{Y}_{32} & \underset{\sim}{Y}_{33} & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & \underset{\sim}{0} & & \cdot & \underset{\sim}{Y}_{L-1,L-1} & \underset{\sim}{Y}_{L-1,L} \\ & & & & \underset{\sim}{Y}_{L,L-1} & \underset{\sim}{Y}_{LL} \end{bmatrix} \tag{9}$$

The external admittance matrix $\underset{\sim}{Y}_{11(e)}$ can be obtained from $\underset{\sim}{Y}_{nn}$ by suppressing the internal nodes layer by layer from the center to the boundary [6]

$$\underset{\sim}{Y}_{L-1,L-1(e)} = \underset{\sim}{Y}_{L-1,L-1} - \underset{\sim}{Y}_{L-1,L} \underset{\sim}{Y}_{L,L}^{-1} \underset{\sim}{Y}_{L,L-1}, \tag{10}$$

$$\vdots$$

$$\underset{\sim}{Y}_{22(e)} = \underset{\sim}{Y}_{22} - \underset{\sim}{Y}_{23} \underset{\sim}{Y}_{33(e)}^{-1} \underset{\sim}{Y}_{32}, \tag{11}$$

$$\underset{\sim}{Y}_{11(e)} = \underset{\sim}{Y}_{11} - \underset{\sim}{Y}_{12} \underset{\sim}{Y}_{22(e)}^{-1} \underset{\sim}{Y}_{21}. \tag{12}$$

The element evaluation is the reversal of the suppressing process. Knowing $\underset{\sim}{Y}_{11(e)}$ from the external measurements, elements will be evaluated by retriving the internal nodes from the boundary to the center. Equation (12) is solved to evaluate the elements in the first layer and $\underset{\sim}{Y}_{22(e)}$ needed for the second layer evaluation. Once $\underset{\sim}{Y}_{22(e)}$ is known, the elements in the second layer and $\underset{\sim}{Y}_{33(e)}$ can be computed by solving (11). In the same way, the remaining layers can be evaluated.

As an example consider the first layer evaluation. Denote admittances of boundary elements by b_i $(i=1,2,\ldots,m_1)$ and the remaining elements of the first layer by g_j $(j=1,2,\ldots,m_1-4)$. Define

$$\underset{\sim}{Z} = \underset{\sim}{Y}_{22(e)}^{-1} \tag{13}$$

and (12) becomes

$$\underset{\sim}{Y}_{11(e)} = \underset{\sim}{Y}_{11} - \underset{\sim}{Y}_{12} \underset{\sim}{Z} \underset{\sim}{Y}_{21}. \tag{14}$$

Note that $\underset{\sim}{Y}_{11}$ is basically a three diagonal matrix containing b_i and g_j only and each entry of matrix $\underset{\sim}{Y}_{12} \underset{\sim}{Z} \underset{\sim}{Y}_{21}$ is the product of three variables. The element evaluation in each layer can be performed in two stages: a set of equations to calculate g_j and z_{pq} are selected from off-diagonal non-zero entries in $\underset{\sim}{Y}_{11(e)}$ (z_{pq} are elements of $\underset{\sim}{Z}$). When g_j and z_{pq} are known, b_i can be obtained directly from the remaining m_1 equations in $\underset{\sim}{Y}_{11(e)}$.

At the first stage, the number of variables is

$$N_1 = m_1 - 4 + m_2(m_2+1)/2 \quad (15)$$

where $m_2=m_1-8$. We formulate the functions $\underset{\sim}{W}$ as

$$w_j = [\ \underset{\sim}{Y}_{11(e)} + \underset{\sim}{Y}_{12}\ \underset{\sim}{Z}\ \underset{\sim}{Y}_{21}\ ]_{k\ell} \quad (16)$$

and solve the nonlinear problem using

$$\underset{\sim}{J}_1\ \Delta\underset{\sim}{X} = -\underset{\sim}{W} \quad (17)$$

where $\underset{\sim}{J}_1$ is the Jacobian matrix of $\underset{\sim}{W}$.

and $$\Delta\underset{\sim}{X} = [\ \Delta\underset{\sim}{G},\ \Delta\underset{\sim}{Z}\ ]^T \quad (18)$$

We find that the system is overdetermined but does not have unique solution the rank of the system is less than N_1. The reason is that the number of variables is increased by m_2 when $\underset{\sim}{Y}_{22(e)}^{-1}$ is replaced by $\underset{\sim}{Z}$. So m_2 equations are added using the zero-row-sum property of the indefinite-admittance matrix. We define functions $\underset{\sim}{U}$ as

$$u_i = \sum_{j=1}^{m_2} y_{22(e)}(i,j) + \sum_{j=1}^{m_1} y_{21}(i,j), \quad i=1,2,\ldots,m_2 \quad (19)$$

The linearized system of equations for $\underset{\sim}{U}$

$$\underset{\sim}{J}_2\ \Delta\underset{\sim}{X} = -\underset{\sim}{U} \quad (20)$$

can be combined with (17) to obtain

$$\underset{\sim}{J}\ \Delta\underset{\sim}{X} = -\underset{\sim}{F} \quad (21)$$

where $$\underset{\sim}{F} = [\ \underset{\sim}{W},\ \underset{\sim}{U}\ ]^T \quad (22)$$

and $$\underset{\sim}{J} = \begin{bmatrix} \frac{\partial \underset{\sim}{W}}{\partial \underset{\sim}{G}} & \frac{\partial \underset{\sim}{W}}{\partial \underset{\sim}{Z}} \\ \frac{\partial \underset{\sim}{U}}{\partial \underset{\sim}{G}} & \frac{\partial \underset{\sim}{U}}{\partial \underset{\sim}{Z}} \end{bmatrix} \quad (23)$$

The new system is consistent because the rank of the augmented Jacobian matrix equals the number of variables, i.e., $\text{rank}(\underset{\sim}{J}')= N_1$. So the values of all variables can be obtained by the iterative process.

The algorithm of the layer by layer method is as follows:

1. Form $\underset{\sim}{Y}_{11(e)}$ from the measurements and determine the number of layers L in the grid model.
2. Calculate the number of external nodes m and the number of variables N and give initial values of the variable vector $\underset{\sim}{X}^0$ for the ℓth layer.
3. Compute $\underset{\sim}{F}^k$, $\underset{\sim}{J}^k$ and $\underset{\sim}{J}'^k$ based on $\underset{\sim}{X}^k$ and evaluate $\text{rank}(\underset{\sim}{J}'^k)$ for the kth iteration. If $\text{rank}(\underset{\sim}{J}'^k) \geq N$, the system is solvable. Otherwise it is not solvable and the process stops.
4. Solve the linear equations $\underset{\sim}{J}^k\ \Delta\underset{\sim}{X}^k = -\underset{\sim}{F}^k$ to get $\Delta\underset{\sim}{X}^k$ and update $\underset{\sim}{X}$ using $\underset{\sim}{X}^{k+1} = \underset{\sim}{X}^k + t^k\ \Delta\underset{\sim}{X}^k$ $(0<t^k\leq 1)$.
5. If $|\ \Delta\underset{\sim}{X}^k\ |$ and $|\ \Delta\underset{\sim}{F}^k\ |$ do not reach the required accuracy, then k=k+1 and go to 3.
6. Calculate b_i after getting g_j and z_{pq}.
7. Store the values of elements on the ℓth layer and compute $\underset{\sim}{Y}_{\ell-1,\ell-1(e)}$ for the next layer from $\underset{\sim}{Z}$.
8. If $\ell=1$, then stop. Otherwise $\ell=\ell-1$ and go to 2.

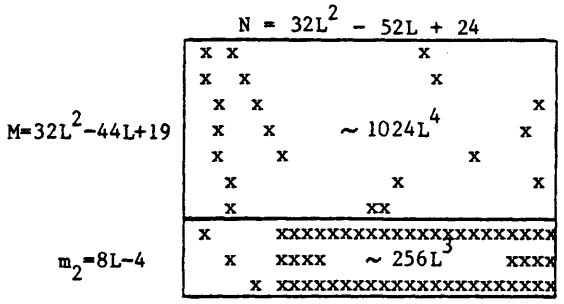


Fig. 2 Dimension and nonzero pattern of $\underset{\sim}{J}$ (23).

let us investigate the size of the Jacobian matrix (Fig. 2). Since each function of $\underset{\sim}{W}$ is the product of three variables, $\underset{\sim}{J}_1$ is a very sparse matrix although it is quite large. The sparse matrix technique will be used to save the memory and speed up the computation. $\underset{\sim}{J}_2$ is composed of two parts: $\partial\underset{\sim}{U}/\partial\underset{\sim}{G}$ and $\partial\underset{\sim}{U}/\partial\underset{\sim}{Z}$. The first part is small and sparse but the second part is dense since

$$\partial u_i/\partial z_{pq} = \partial[\sum_{j=1}^{m} y_{22(e)}(i,j)]/\partial z_{pq}$$

$$= -\sum_{j=1}^{m} y_{22(e)}(i,p)\,y_{22(e)}(q,j) \quad (24)$$

The size of this part is nearly equal to $256L^3$ when L is very large. Comparing with the direct method the size of the dense part is substantially reduced from $\sim L^4$ to $\sim L^3$.

VI. RESULTS AND DISCUSSION

This layer by layer method was implemented and tested using computer-generated data for several admittance patterns. For each pattern, $\underset{\sim}{Y}_{11(e)}$ was computed based on the actual values of elements. At each iterative step, the linear system (21) was solved by Gauss-Jordan elimination using the maximum pivot strategy.

Example

As an example consider the grid model shown in Fig. 3. It can be divided into two layers: b_i and g_j belong to the first layer; c_k belong to the second layer.

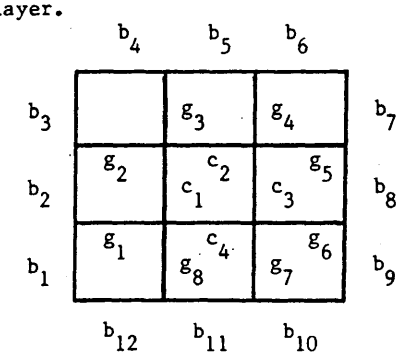


Fig. 3 The grid model for the example.

There are total 18 variables in the first stage: $\underset{\sim}{X} = [\, g_1, g_2, \cdots, g_8, z_{11}, z_{12}, x_{13}, z_{14}, z_{22}, \cdots, z_{44} \,]^T$. 24 equations w.r.t. the entries in $\underset{\sim}{Y}_{11(e)}$ represented by the sign x (see Fig. 4(a)) are used to define functions $\underset{\sim}{W}$,

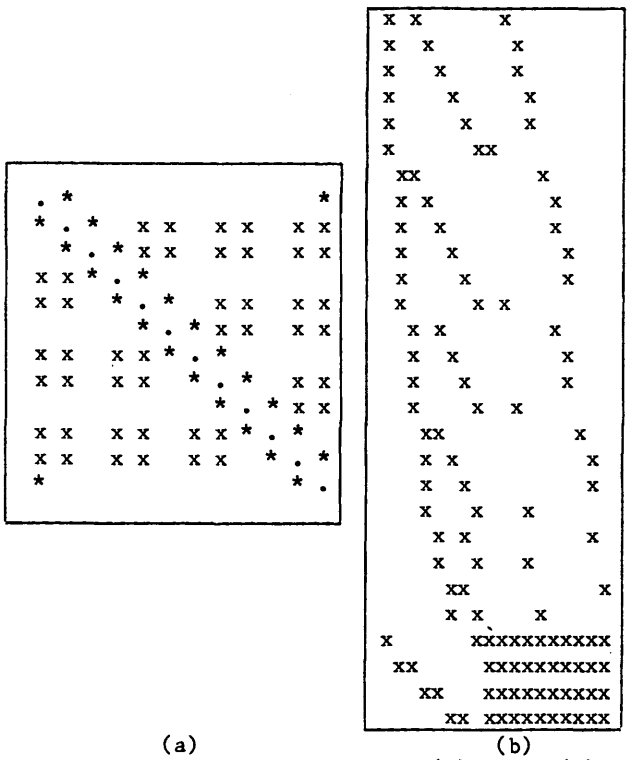
$$w_i = [\underset{\sim}{Y}_{11(e)}]_{k\ell} + g_j z_{pq} g_r \quad i=1,2,\ldots,24 \qquad (25)$$

and rank($\underset{\sim}{J}_1'$) = 14. So additional 4 functions $\underset{\sim}{U}$ are as follows:

$$\begin{aligned} u_1 &= \sum_{j=1}^{4} y_{22(e)}(1,j) - g_1 - g_8 \\ u_2 &= \sum_{j=1}^{4} y_{22(e)}(2,j) - g_2 - g_3 \\ u_3 &= \sum_{j=1}^{4} y_{22(e)}(3,j) - g_4 - g_5 \\ u_4 &= \sum_{j=1}^{4} y_{22(e)}(4,j) - g_6 - g_7 \end{aligned} \qquad (26)$$

The combined system of equations formulated as (21) is consistent. g_j and z_{pq} can be obtained by the iterative process and results are listed in Table. Nine iterative steps were needed to meet requirements of the 0.1% precision. The non-zero pattern of the Jacobian matrix is shown in Fig. 4(b). Density of $\underset{\sim}{J}_1$ is 15% and the size of the dense part is 4x10.

According to the algorithm in Section III, b_i can be obtained using 12 equations w.r.t. the entries in $\underset{\sim}{Y}_{11(e)}$ represented by the sign * (Fig. 4 (a)) and c_k can be calculated after inverting $\underset{\sim}{Z}$.



(a) (b)

Fig. 4 Nonzero Entries of $\underset{\sim}{Y}_{11(e)}$ (a) and $\underset{\sim}{J}$ (b).

Table. Results

elements	actual X values	initial X^0 values	computed X^9 values
g_1	1.000	1.500	1.000
g_2	1.000	1.500	1.000
g_3	1.000	1.500	1.000
g_4	1.000	1.500	1.000
g_5	1.000	1.500	1.000
g_6	1.000	1.500	1.000
g_7	1.000	1.500	1.000
g_8	1.000	1.500	1.000
z_{11}	0.292	0.233	0.292
z_{12}	0.083	0.100	0.083
z_{13}	0.043	0.067	0.043
z_{14}	0.083	0.100	0.083
z_{22}	0.292	0.233	0.292
z_{23}	0.083	0.100	0.083
z_{24}	0.042	0.067	0.042
z_{33}	0.292	0.233	0.292
z_{34}	0.083	0.100	0.083
z_{44}	0.292	0.233	0.292

V. CONCLUSION

Results of our simulation study indicate that it is feasible to estimate the elements values in a large resistive network from the external measurements. By using the layer by layer method, the size of the dense part in the Jacobian matrix is decreased from L^4 to L^3. Thus the computational effort is largely decreased. Since each layer evaluation depends on the results obtained from the previous layer, a number of iterative steps is needed at each layer to ensure accuracy. It is a significant improvement for the ICT application. This evaluation method will be developed by improving the numerical procudure to reduce error and by combining parallel processing with decompositon and sparse techniques to increase efficiency. The method will be tested on experimental data.

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