Agnieszka Kończykowska*
Centre National d'Etudes des Télécommunications, France
Janusz Starzyk
Warsaw Technical University, Poland

COMPUTER ANALYSIS OF LARGE SIGNAL FLOWGRAPHS BY HIERARCHICAL DECOMPOSITION METHOD

1. INTRODUCTION

Signal flowgraph method of linear network analysis used to be an attractive and convenient instrument only in the case of very small networks. Elaboration of computer programs /e.g. SNAP[4], NASAP [3]/ based on signal flowgraph representation of electronic networks has allowed to estimate real possibilities of this method. In paper [4] the authors have affirmed that "results from SNAP indicate that for reasonable computing times, a 15-node, 30-branch network is about the maximum that can be handled". The restrictions on direct topological analysis methods follow from exponential growth of the number of terms in the Mason's formula for the network determinant as a function of a graph nodes number. So no matter how efficient technique will be used we shall not be able to analyse directly large signal flowgraphs.

On the other hand various analysis programs based on the numerical methods have been elaborated recently. They allow to obtain symbolic functions of linear active networks. An application of sparse matrix technique makes them very efficient and fast. An application of diacoptic resulted in further improvement of the numerical methods. But even then they still do not ensure suffiecient accuracy in some practical cases and still are not efficient enough for solving such network design problems as centering and tuning. Many other computational techniques used in computer aided analysis of electronic circuits can be improved together with the method

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*on leave from Warsaw Technical University of solving the linear equations.

2. METHOD OF HIERARCHICAL ANALYSIS

The aim of this paper is the presentation of the computer implementation of the new topological signal flowgraph analysis concept as well as some computational results and remarks connected with the program to be discused. This realisation is based on the recent papers of the authors [5,9].

Recently there has been published some papers on topological signal flowgraph analysis in an attempt to avoid the term cancelation which can occur in Mason's formula for the graph determinant [6,7]. New signal flowgraph topological formulas for the graph determinant have been proved. In these formulas no term cancelation occurs. Simple criterions has been established which permit to eliminate the duplicate terms during the generation routines. But still the number of terms to be generated remains the exponential function of the number of graph nodes and that may cause problems when greater networks will to be handled.

In [5,9] the concept of hierarchical decomposition has been presented. This method allows to obtain the involutive dependance for analysis time in a function of graph nodes number. For quasi-optimal partitions the exponent of the power is about 2.

A signal flowgraph can be decomposed in one of three manners /see Fig.1/:

I Nodes decomposition. A graph is divided into edge disjoint subgraphs, the sum of which contains all nodes of a graph. Common nodes of different subgraphs are called block nodes.

II Edge decomposition. A graph is divided into node disjoint subgraphs by extraction of a subset of the set of edges.

III Hybrid decomposition - a combination of two previous decompositions.

In our presentation the edge decomposition and Coates representation of electrical networks 2 will be used.

Let us cosider a flowgraph which can be a graph of an electrical network or a graph of any system of linear equations as well.

We denote:

$G(V,E)$ - a flowgraph with V - set of vertices and E - set of edges,

$W = \langle W^1, W^2 \rangle$ set of vertices such that $W^1, W^2 \subset V$ and card W^1 = card $W^2 = k$.

Definition 1. We call a general k-connection of type C_W any spanning subgraph of G such that c_W forms in G k separated paths with W^1 set of begginings and W^2 set of ends plus a set of loops disjoint with these paths.

If a function w(e) is a transmittance of weight function of graph edges, then a weight of a k-connection is defined as follows

$$f(c_W) = (-1)^n \sum_{e \in c_W} w(e) \qquad /1/$$

where n denotes number of loops in c_W.

Formulas for a determinant and a cofactor of a flowgraph calculated with an aid of Mason's rule are following

$$\Delta = F(C) = \sum_{c \in C} f(c) \qquad /2/$$

$$\Delta_{ij} = F(C_W) = \sum_{c_W \in C_W} f(c_W) \qquad /3/$$

where $W = \langle i,j \rangle$.

Let us denote:

E_{cut} - cutset of a graph G;

V_{cut} - set of vertices incident with E_{cut} edges, block vertices;

$G_1(E_1, V_1)$, $G_2(E_2, V_2)$ - simple edge decomposition into two disconnected graphs obtained from G after removing edges E_{cut}.

Definition 2. Complete graph spanned on block vertices incident with edges E_i is called a substitute graph for a subgraph G_i and denoted G_i^s .

Definition 3. Graph $G^d = G_1^s \oplus G_2^s \oplus G_{cut}(E_{cut}, V_{cut})$ is called a substitute graph for a simple decomposition.

Definition 4. We consider a graph G^d with ordered set of nodes /e.g. integer positive numbers/. k-connection p_W of that graph is called properly ordered one of the type C_W if

a/ $p_W \in C_W$,

b/ set of paths $Z_i = p_W \cap G_i^s$

- has no longer than one element paths
- $\forall \langle v_1, w_1 \rangle , \langle v_2, w_2 \rangle \in Z_i \; (v_1 < v_2 \Rightarrow w_1 < w_2)$

The set of properly ordered k-connections of type C_W we denote P_W, and W_i are sets of border vertices of G_i^s edges.

Example 1. Let us consider a simple decomposition presented in Fig.2. The set of properly ordered 0-connections of the graph G^d is $C = \{ \{\langle 1,1 \rangle , \langle 2,2 \rangle , \langle 3,3 \rangle , \langle 4,4 \rangle , \langle 5,5 \rangle , \langle 6,6 \rangle \} , \{ \langle 1,2 \rangle , e_2, e_3, \langle 5,4 \rangle , \langle 3,3 \rangle , \langle 6,6 \rangle \} , \{ \langle 2,1 \rangle , e_1, e_4, \langle 3,6 \rangle , \langle 4,4 \rangle , \langle 5,5 \rangle \} , \{ e_1, e_2, e_3, e_4, \langle 3,4 \rangle , \langle 5,6 \rangle \} , \{ \langle 1,1 \rangle , e_3, e_4, \langle 6,5 \rangle , \langle 4,4 \rangle , \langle 3,3 \rangle \} , \{ \langle 2,2 \rangle , e_1, e_2, \langle 3,4 \rangle , \langle 5,5 \rangle , \langle 6,6 \rangle \} \}$.

Theorem 1. In a case of simple edge decomposition the determinant of a flowgraph can be calculated from the formula

$$\Delta = \sum_{c \in C^P} (-1)^{n_c} \; f(E_{cut} \cap c) F(C_{W_1}) \; F(C_{W_2}) \qquad /4/$$

where

$$F(C_{W_i}) = \sum_{c_{W_i} \in C_{W_i}} f(c_{W_i})$$

n_c - number of loops in c.

For a flowgraph with different edge transmittance the formula /4/ has no reducible terms.

Analogous formulas can be formulated for flowgraph cofactors.

When a simple decomposition is applied many times towards the subgraphs obtained during previous steps we deal with the hierarchical decomposition. The sequence of simple bisections appearing in the hierarchical decomposition can be represented as a tree of decomposition which vertices denote either substitute graph when further partition occurs or final subgraphs of a decomposed graph. /Fig.3/.

3. MAIN CHARACTERISTICS OF THE PROGRAM OF ANALYSIS

The FORTRAN program FANES based on presented results has been elaborated. As the input data the decomposed signal flowgraph should be introduced. But as none restrictions are imposed on the partition procedure the decomposition can be carried out automatically. Any efficient algorithm for graph partition can be used. Many of them have been recently elaborated /e.g. see [8]/. The program realizes formula /4/ in the hierarchical structure. 5 polynomials of the network are computed in fully symbolic form and presented as polynomials of Laplace variable s. We start analysis by determining properly ordered connections of the substitute graph from the level 1. On each stage when during the analysis we deal with the substitute graph we continue procedure on the lower level according to the structure of the hierachical decomposition tree and find k-connections of the type determined on the previous level /theorem 1/. Computations are organized in the way that maximal number of incidence matrices treated simultaneously is not greater than number of levels in the tree of decomposition.

The edge decomposition and the notion of properly ordered k-connection enables us to such organisation of the program that connections are determined in groups which reduces the number of transfers in the hierarchical structure and as result the time of calculations.

If we analyse a signal flowgraph of an electrical twoport 5 characteristic polynomials of the network can be obtained from the formulas

$$k_{iu} = N/F(C) \qquad /5/$$

$$k_{uu} = N/F(C_a) \qquad /6/$$

$$k_{ui} = N/\left(F(C_{ab}) - F(C_{ac}) - F(C_{ae}) + F(C_{af})\right) \qquad /7/$$

$$k_{ii} = N/\left(F(C_b) - F(C_c) - F(C_e) + F(C_f)\right) \qquad /8/$$

$$N = F(C_d) - F(C_g) \qquad /9/$$

where indices represent additional edges shown in Fig.4.

The formula /4/ has no reducible terms if all flowgraph edge transmittances are different. It is not the case with various flowgraph representations of electrical networks. This is the cause of reducible terms in Mason's formula. We shall show now how strong is the dependance of the number of generated terms on the variables choosen as vertices for the signal flowgraph.

On purpose to profit the automatic formation of a flowgraph for an electrical network, two models of oneport have been examinated /see Fig.5/. As an example let us consider a passive ladder filter with 11 admitance branches and 7 nodes. In the table I the results of computation for both representations are compared.

All computation of FANES have been executed on CDC Cyber 73.

TABLE I

characteristic polynomials	Representation 5b no of edges 44, no of vertices 7		Representation 5c no of edges 49, no of vertices 16	
	N^o of terms	Execution time(sec)	N^o of terms	Execution time(sec)
n_{in}	797	.05	53	.16
n_{uu}	549	.04	133	.19
n_{ui}	248	.03	80	.13
n_{ii}	360	.04	32	.11
n	4	.02	4	.03

It is evident that with the 5c representation number of generated terms is much reduced in comparision with 5b graph. In the case of the analysed filter the column 3 gives the exact number of terms for characteristic polynomials so no reduction occurs. The inconvenience of the 5c representation is that the number of edges and vertices is greater than for 5b which in spite of less term number causes longer computation time.

To give the idea of the increase of computation time with the size of analysed graph, we shall present some examples:

a/ a flowgraph of an equalizer with 180 edges and 24 nodes - time of computation of semisymbolic form of voltage transfer function 11 sec

b/ a calculation of semisymbolic form of a determinant of matrix (72×72) with 204 non-zero elements - time 10 sec

c/ time of generation of a transfer function of a resistive ladder in comparison with other symbolic or topological methods is shown in a table II. /Compare [10]/.

TABLE II

Number of nodes including ref.node	Execution time in sec			
	FANES	SNAPEST	NAPPE	SNAP
11	.10	.298	1.23	3.27
13	.11	.370	1.67	19.8
15	.14	.463	2.35	140.*
17	.19	.567	3.12	16 min* *estimated

4. CONCLUSIONS

The paper presents computer implementation and short review of an efficient topological method for generating symbolic form of transfer function for large networks. The method shown in the paper presents usual advantages of topological analysis like e.g.:

1/ High accuracy of computations
2/ Reduction of the total computation time when the big number of frequency points or of changes of elements values is requested
3/ Simplification of sensitivity analysis
4/ Possibility of accomplishment of approximative symbolic analysis
5/ Possibility of generation of macromodels transfer functions
6/ Simplification of time-domain analysis
7/ Easier stability analysis and determination of zeros and poles of transfer functions
8/ Easier tolerance, centering and statistical analysis.

The presented method can still be improved. At present the modified hierarchical analysis method is elaborated. The analysis will be started now on the lower level of decomposition. Estimations of the method permit to affirm that new approach can result in diminution of computation time with only slight increase of the computer memory demand.

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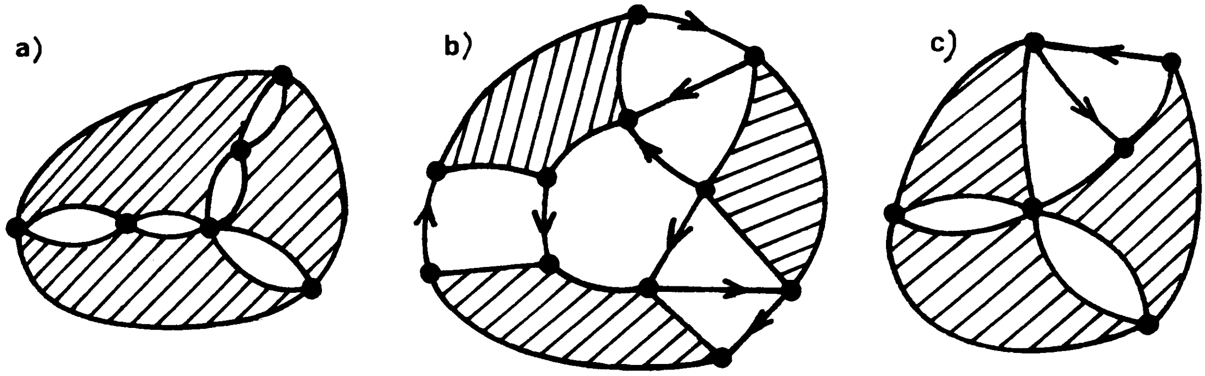

Fig.1. a/ Node docomposistion b/ Edge decomposition
c/ Hybrid decomposition

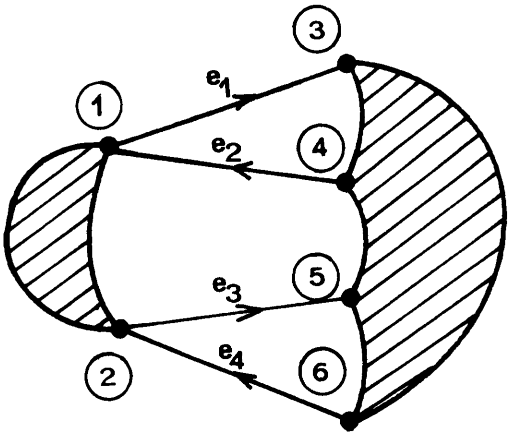

Fig.2. Simple edge decomposition

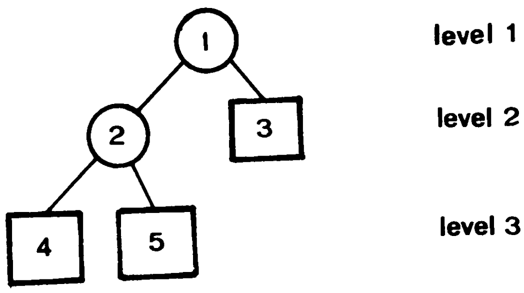

Fig.3. Tree of hierarchical decomposition