AN OPTIMIZATION APPROACH TO FAULT LOCATION IN ANALOG CIRCUITS

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This paper addresses the problem of fault location in analog circuit under voltage measurements. With a sufficient number of voltage measurements the least squares criterion is efficiently employed to identify faulty elements and calculate their values. The L_1 norm is utilized in isolating the most likely faulty elements under a given and limited number of measurements. Examples demonstrating the effectiveness of the proposed methods in locating the faulty elements are presented.

1. INTRODUCTION

In recent years a considerable amount of effort has been directed towards techniques by which the problem of fault location can be approached. The main objective of fault diagnosis is to detect the faulty elements in the circuit.

By a fault we mean in general any change in the value of the element with respect to its nominal value which can cause the failure of the circuit performance.

Fault location can be done by the method which identifies all element values [1], and then compares the nominal and actual values. The fault evaluation is thus being done simultaneously with the fault detection. However, this parameter identification approach needs a sufficient number of independent voltage measurements and is computationaly demanding.

With a limited number of independent voltage measurements fault location is carried out by identifying the faulty components under the assumption that there are very few of them and values of remaining components are within their tolerances [2].

In this paper we discuss two efficient approaches for fault location.In the first one, we formulate the fault location problem as an unconstrained optimization problem with least squares objective function. The outcome of this optimization problem is a complete identification of all network elements and consequently location of the faulty elements that drift out of their tolerances. In the second approach, we utilize the L_1 norm in estimating the most likely faulty elements. At the end we give examples demonstrating the capabilties of both methods in the detection of faulty elements.

2. PARAMETER IDENTIFICATION

The parameter identification problem can be formulated in the following way. Given a sufficient number of independent voltage measurements greater than the number of the elements of the faulty network, it is required to identify all the elements of the network. The circuit under test is assumed to be of known topology. In general a subset of network nodes are accessible nodes (test nodes) where voltages and/or currents can be applied and/or measured and the rest of the nodes are internal nodes (inaccessible nodes), at which neither voltages nor currents can be applied or measured. It is further assumed that the nominal element values are known.

One then desires to identify the elements of the circuit by performing measurement at the test nodes. The faulty elements are consequently identified by determining which circuit's element values fall outside the tolerance margins.

In what follows we present a simple formulation that employs the least squares criterion to identify all network elements.

First of all we simulate the network with nominal element values to get the computed voltage vector $\underset{\sim}{V}^c$. Then we measure voltages at the test nodes to get the voltage measurement vector $\underset{\sim}{V}^m$. This will enable us to construct the following objective function

$$F = \sum_{i=1}^{n_x} \sum_{j \in I_m} (V_{ji}^m - V_{ji}^c)^2 \qquad (1)$$

where I_m is the set of accessible nodes, and n_x is the number of independent excitations.

It should be noted that $\underset{\sim}{V}^m$ is a constant vector. It is clear that the minimization of the above objective function will yield to the identification of all the element values of the circuit under test. The objective function F is considered as an unconstrained optimization problem, which can be solved very efficiently utilizing appropriate optimization techniques [3], to find the values of all elements.

The question that arises immediately is how to choose the nodes for independent current excitations (excitation nodes) to obtain independent measurements especially for large size networks. Different approaches have been suggested to partially answer this question [4,5].

The selection of the excitation nodes can be based on a rank test. Similar to the work presented in [4], it can be shown that for the local uniqueness of the solution we must select the set of the excitation nodes such that the matrix $\underset{\sim}{B}_{nxn}$ is of full rank, where

$$\underset{\sim}{B}_{nxn} = \underset{\sim}{A}_{n\,x\,m_e} \cdot \underset{\sim}{A}^T_{m_e x\,n} \qquad (2)$$

$\underset{\sim}{A}_{nxm_e}$ is the matrix of the derivatives of the voltages measured with respect to the elements of the circuit at each excitation n is the number of network elements, and m_e is the number of accessible nodes multiplied by the number of excitations.

3. FAULT LOCATION USING THE L_1 NORM

In this section fault location is discussed with a limited number of independent voltage measurements. Obviously the identification of all network elements in this case is impossible. The fault location is done by identifying the faulty elements under the realistic assumption that they are very few in number and the relative changes in their values are significantly larger than in the non-faulty ones.

The fault diagnosis problem is generally formulated as follows [6]. Given $\Delta \underset{\sim}{V}^m$ as the m - dimensional vector of the changes of the measured voltages from their nominals, determine the associated n - dimensional vector of the changes of the elements from nominal values $\Delta\underset{\sim}{\phi}$ where

$$\Delta \underset{\sim}{V}^m = \underset{\sim}{S}\, \Delta\underset{\sim}{\phi} \quad , \qquad (3)$$

and $\underset{\sim}{S}$ is the m x n sensitivity matrix. Equation (3) is an underdetermined system of equations in the parameters $\Delta\phi$. A least one objective function is utilized to estimate the most likely faulty elements. The problem is formulated as a linear optimization problem in the following way. Find the vector $\Delta\underset{\sim}{\phi}$ which minimizes the objective function,

$$F_1 = \sum_{i=1}^{n} \left| \frac{\Delta\phi_i}{\phi_i^o} \right| \qquad (4a)$$

subject to

$$v_j^m - v_j^c = 0 \ , \ j \in I_m \qquad (4b)$$

where voltages v_j^m are measured under a single excitation. The normalization with respect to the nominal element values ϕ_i^o is recommended especially when the nominal values of the elements are varied over a wide range.

This objective function penalizes the non-zero elements in the solution exploiting the properties of the L_1 norm. A nonzero element of $\Delta\phi$ implies failure of the element associated with it.

It is the authors' experience that the discontinuities of the derivative of F_1 at $\Delta\phi_i = 0$, may cause some practical problems when using general optimization algrithms that can not deal with the L_1 norm objective function.

In order to avoid these problems we define a 2n - dimensional vector $\underset{\sim}{X} > 0$ such that,

$$x_i = \Delta\phi_i \text{ and } x_{n+i} = 0 \qquad \text{if } \Delta\phi_i > 0$$

$$x_i = 0 \text{ and } x_{n+i} = -\Delta\phi_i \qquad \text{if } \Delta\phi_i < 0 \qquad (5)$$

where $\Delta\phi_i = x_i - x_{n+i}$

the optimization problem presented in (3) can be restated as follows,
Find the vector $\underset{\sim}{X}$ which minimizes

$$F_2 = \sum_{i=1}^{2n} x_i \qquad (6a)$$

subject to

$$v_j^m - v_j^c = 0 \ , \quad j \in I_m \qquad (6b)$$

$$\text{and} \qquad \underset{\sim}{X} > \underset{\sim}{0}$$

Although the number of variablies is doubled, the problem described by equations (6) is easier to be solved than the one described by (4) and usually leads to more accurate results.

4. EXAMPLES

To illustrate the methods two examlpes were run on the CDC 170/815 system and results obtained are presented in Tables I and II.

Example 1

Consider the resistive network shown in Fig 1. which was originally considered by Bandler et al [2]. The nominal values of elements $G_i = 1$ and tolerances $\varepsilon_i = \pm 0.05$, i=1,2,....,20. The outside nodes are assumed to be accessible with node 12 taken as the reference node. Nodes 4,5,8,9 are internal nodes where no measurements can be applied.

Two faults are assumed in the network elements G_2 and G_{18}. For case 1, we solved the unconstrained optimization problem (1) with independent excitations at nodes 1,6 and 7. For Case 2, we consider the

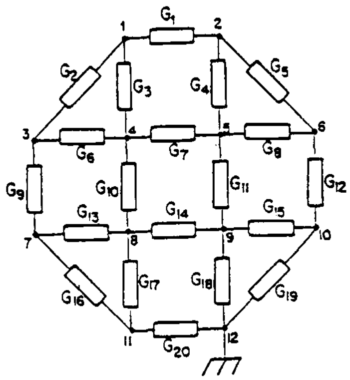

Fig. 1 The resistive network

optimization problem described by (6) with a single excitation at node 1. In both cases the faulty elements have been correctly identified as we can see from Table I.

TABLE I
RESULTS FOR EXAMPLE 1

Element	Percentage Deviation		
	Actual	Case 1	Case 2
G_1	-2.0	0.0	-1.0
G_2	-50.0	-48.0	-50.2
G_3	4.0	5.1	0.0
G_4	-3.0	-4.0	0.0
G_5	-5.0	-2.0	-2.5
G_6	-1.0	2.0	0.0
G_7	2.0	4.0	0.0
G_8	5.0	1.5	0.1
G_9	2.0	1.0	0.0
G_{10}	-2.0	-1.0	0.0
G_{11}	4.0	1.0	1.2
G_{12}	1.0	2.0	0.0
G_{13}	-1.0	1.5	0.0
G_{14}	-2.0	-3.0	0.5
G_{15}	2.0	1.5	0.0
G_{16}	-4.0	-3.8	0.0
G_{17}	2.0	0.0	1.3
G_{18}	-50.0	48.2	-49.2
G_{19}	-2.0	-3.9	-1.0
G_{20}	-4.0	-4.6	-2.0

Example 2 [7]

Consider the single stage transistor amplifier shown in Fig.2 with its equivalent circuit in Fig.3. Three faults are assumed in C_1, r_π and g_m. We excite the circuit at node 1 with angular frequency ω = 0.01 rad/sec and simulate voltage measurements at the accessible nodes 1,2,4,5 and 6. For the sake of comparison we have quoted the results presented in [2], which are referred to as case 1* in Table II. For case 2 we again apply the optimization problem described by (6). In both examples the obtained results represent an improvement over those in [2], with the estimated changes in the faulty elements approaching their true values. For example, using our approach g_m was detected as a faulty element where as in the method described in [2], g_m has not been detected as a fault.

It is obvious that if we use more than one excitation for case 2, then we add more information about the network. Accordingly the problem becomes less underdetermined and the proposed method is expected to give better results.

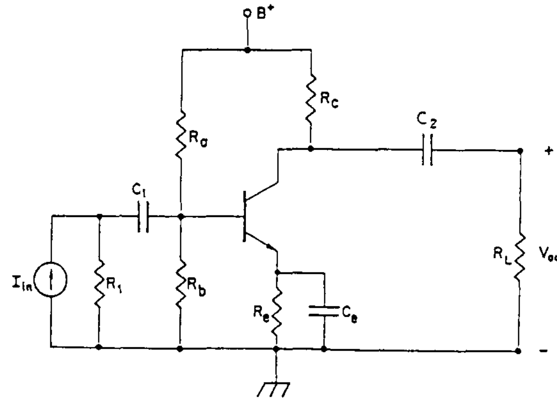

Fig. 2 The transistor amplifier circuit

TABLE II.
RESULTS FOR EXAMPLE 2

Element	Percentage Deviation		
	Actual	Case 1*	Case 2
C_1	-50.00	-48.68	-48.23
R_s	2.56	0.0	1.73
r_x	2.00	0.0	0.52
r_π	66.66	-12.93	-22.36
c_π	-6.66	0.0	0.0
c_μ	-4.00	-0.32	0.0
g_m	-50.00	0.0+	-46.34
R_c	-2.00	-1.36	-1.28
C_2	-5.00	-0.65	-1.37
R_L	3.00	1.43	0.78
R_2	-1.96	4.97	-0.42
C_e	-5.00	0.0	0.0
R_1	0.50	-1.43	0.54

+ A faulty element has not been detected

5. CONCLUSIONS

Two approaches for fault location in analog circuits have been presented and computational results discussed. They depend upon voltage measurements using current excitations. The least squares criterion is effectively employed to identify all element values and consequently the faulty elements are located. With the practical restriction on a number of independent voltage measurements we exploited the properties of the L_1 norm in estimating the most likely faulty elements. The proposed modification of the least one objective function allows us to obtain improvement over the fault location technique pre-

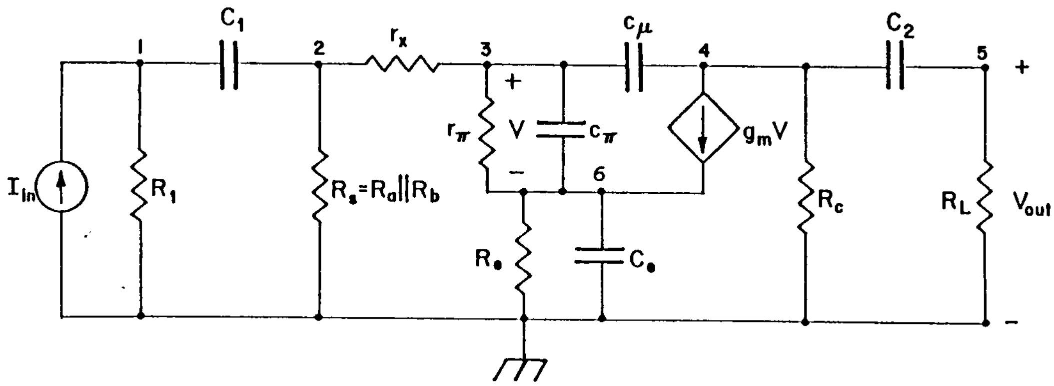

Fig. 3 The amplifier equivalent circuit.

sented in [2], giving better estimates for element values. Illustrative examples which clearly indicated the efficiency of the proposed approaches are presented .

REFERENCES

[1] T.N. Trick, W. Mayeda and A.A. Sakla, "Calculation of parameter values from node voltage measurements", IEEE Trans.Circuits and Systems, vol. CAS-26,pp. 466-474,1979.

[2] J.W. Bandler, R.M. Biernacki, A.E. Salama and J.A. Starzyk, "Fault isolation in linear analog circuits using the L_1 norm", Proc. IEEE Int. Symp. Circuits Systems(Rome,Italy),pp. 1140-1143, 1982.

[3] J.W. Bandler and W.M.Zuberek, "A FORTRAN package for minimization with general constraints", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-294, 1982.

[4] N. Sen and R. Saeks, " Fault diagnosis for linear systems via multifrequency measurements", IEEE Trans Cricuits and Systems. vol. CAS-26,pp.457-465, 1979.

[5] V. Visvanathan and A.Sangiovanni-Vincentelli, "Diagnosability of non-linear circuits and systems - part I: the dc case", IEEE Trans. Circuits and Systems , vol. CAS-28, pp.1093-1102, 1981.

[6] H.M. Merrill, "Failure diagnosis using quadratic programming",IEEE Trans. Reliability, vol. R-22, pp.207-213, 1973.

[7] H.S.M. Chen and R. Saeks, "A search alogrithm for the solution of the multifrequency fault diagnosis equations", IEEE Trans. Circuits and Systems, vol. CAS-26, pp. 589-594, 1979.