A TEST GENERATION ALGORITHM FOR PARAMETER IDENTIFICATION OF ANALOG CIRCUITS

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Abstract

An algorithm is developed to generate tests which are sufficient for identification of component values of linear analog circuits. The method is based on topological properties of Coates flow graphs. Although the algorithm does not pretend to provide the best possible solution it gives reasonable and satisfactory results which agree with known examples.

1. INTRODUCTION

Recently, there has been growing interest in fault diagnosis and automatic testing techniques for analog circuits /e.g., [1-11]/. One of the possible approaches to fault location makes use of the identification of all component values [1, 3-8]. Another one is based on fault simulations and constructing a fault dictionary /e.g., [9]/. However, this is not particularly suitable for soft fault location. In some other approaches it is assumed that only a few faults occures which may be or may not be true. The identification of all component values has the advantage that no assumptions on faults are necessary. However, it is limited by the accessibility of the terminals; almost all terminals have to be accessible. On the other hand, the identification of actual component values has some other applications, e.g., in deterministic tuning e.g., [11, 12]

The solvability of the all parameter identification problem was initiated by Berkowitz [1]. His approach was mostly based on current measurements. The use of voltage measurements only was investigated by several other authors [3-5, 10]. They formulated appropriate equations which could be used to calculate component values based on sufficient number of independent measurements. How to arrange for such independent measurements has been the main goal of further investigations [10].

This paper extends the reults obtained by Biernacki and Starzyk in [10] where sufficient test conditions were formulated. The extension is to provide an efficient algorithm for test generation and to express the conditions in terms of topological properties of networks. The Coates flow graph representation is used.

2. THEORETICAL BASIS

Consider a linear network which can be described by nodal equations. Tests under consideration consist of nodal voltage measurements when different current excitations are applied and are assumed to be performed at a single frequency. Mathematically, the ith test is defined as a vector V_n^i of all nodal voltages measured in the presence of unit current excitation between the ith and the datum nodes. The square matrix

$$V_t = \left[V_n^1 \; V_n^2 \dots V_n^N \right] \qquad /1/$$

where N+1 is the number of all nodes, is said to be the matrix of voltage tests. According to [10] the nodal admittance matrix Y_n could be obtained as

$$Y_n = V_t^{-1} \qquad /2/$$

if all tests i=1, 2, ..., N were performed. However, in order to shorten the time of testing, we would like to find a reasonably small subset of these tests which could be sufficient to find Y_n. The knowledge of Y_n enables us, under minor assumptions, to calculate all component values, since the network topology is known.

In [10] it was shown how to check whether a chosen subset of tests $a \subset \{1,2,\dots,N\}$ was sufficient for the identification. Unfortunately, it required to find an appropriate sequence of cut-sets. It is seen that an extraordinary number of combinations can be involved, even for networks of medium size. That is why, the situation was reversed: first, we chose a sequence of cut-sets and then we tried to find the smallest number of tests. Of course, the solution might not be optimal. In this paper we follow the main idea of this approach, but we try to determine a "good" sequence of cut-sets, and also, to get some information on necessary tests which are independent of the choice of a sequence.

Equation /2/ can be rewritten in the form

$$V_t^T Y_n^T = 1 \, . \qquad /3/$$

Consider the product of V_t^T and the jth column of Y_n^T. We have

$$V_t^T y_j = \begin{bmatrix} V_n^{1\,T} \\ V_n^{2\,T} \\ \vdots \\ V_n^{N\,T} \end{bmatrix} \begin{bmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{nj} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \leftarrow \text{jth row} \\ \\ \\ \\ \end{matrix} \qquad /4/$$

Assume, that only some elements of y_j, say the elements with first indices from the set $b_j = \{j1, \dots, jk\}$, are unknown. Therefore, taking the known terms from the left hand side to the right hand side of /4/ we can modify the equation as

$$V_t^T y_{bj} = \begin{bmatrix} V_n^{1\,T} \\ V_n^{2\,T} \\ \cdot \\ \cdot \\ \cdot \\ V_n^{N\,T} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ y_{j1j} \\ \vdots \\ y_{jkj} \\ 0 \end{bmatrix} = \begin{bmatrix} I_{1j} \\ \vdots \\ \\ I_{Nj} \end{bmatrix} . \qquad /5/$$

In order to determine the elements $y_{j1j}, \dots, y_{jkj}$ we can solve a subsystem of /5/

$$V_t^T [a_j | b_j] \begin{bmatrix} y_{j1j} \\ \vdots \\ y_{jkj} \end{bmatrix} = \begin{bmatrix} I_{i1j} \\ \\ I_{ikj} \end{bmatrix} . \qquad /6/$$

where the equations $a_j = \{i1, \dots, ik\}$ are chosen in such a way that the square submatrix $V_t^T[a_j|b_j]$, obtained as intersection of a_j rows and b_j columns, is nonsingular. See Fig. 1 for illustration.

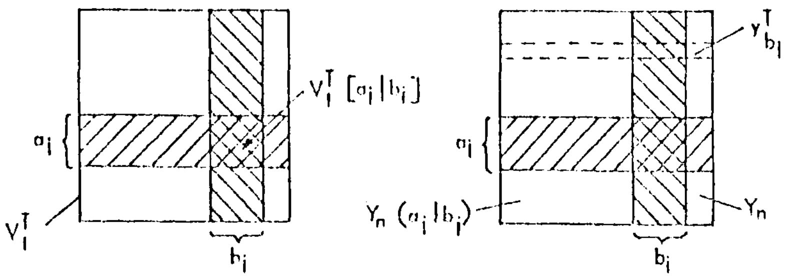


Fig. 1. Illustrations of Eq. /6/ and Theorem 1.

Based on Eqns. /2/ and /6/ the following theorem can be formulated.

Suppose, we are given a sequence of sets b_j, $j = j_1, \dots, j_N$ which corresponds to a sequence of cut-sets of the current graph of the network.

Theorem 1 [10] Tests $a \subset \{1, 2, \dots, N\}$ are sufficient for the identification of all elements of Y_n if and only if

$$\forall b_j \quad \exists a_j \subset a \quad \det Y_n(a_j|b_j) \neq 0 . \qquad /7/$$

where

$$\mathrm{card}(a_j) = \mathrm{card}(b_j) \qquad /8/$$

and $Y_n(a_j|b_j)$ denotes the submatrix of Y_n obtained by removing a_j rows and b_j columns /see Fig. 1/.

As a consequence of /8/ we have the following corollary.

Corollary 1

$$\mathrm{card}(a) \geqslant \max_j \mathrm{card}(b_j) . \qquad /9/$$

It is seen from /9/ that the way of choosing the sequence of b_j is crucial for the minimization of the number of sufficient tests.

Now, in order to characterize tests a_j feasible for a given b_j we consider topological equations for the nodal admittance matrix

$$Y_n = A_- Y A_+^T , \qquad /10/$$

where the ij^{th} element A_- is equal to 1 if the j^{th} edge is directed towards the i^{th} vertex, otherwise zero, and the ij^{th} element of A_+ is equal to 1 if the j^{th} edge is directed away from the i^{th} vertex, otherwise zero.

The submatrix of Y_n obtained by removing b_j columns can be expressed as

$$Y_n(\cdot|b_j) = A_- Y A_+'^T , \qquad /11/$$

where A_+' is obtained from A_+ by removing b_j rows. In Coates graphs, this corresponds to deleting all the edges outgoing from b_j vertices.

Similarly

$$Y_n(a_j|b_j) = A_-' Y A_+'^T , \qquad /12/$$

where A_-' is obtained from A_- by removing a_j rows. In Coates graph, this corresponds to deleting all the edges incoming to a_j vertices.

In order to have a nonzero value of $\det Y_n(a_j|b_j)$ the submatrix $Y_n(a_j|b_j)$ must not contain any zero-row or zero-column. Thus, after deleting the edges mentioned there should exist appropriate edges in the remaining Coates graph, as is illustrated in Fig. 2. N denotes the set of all vertices.

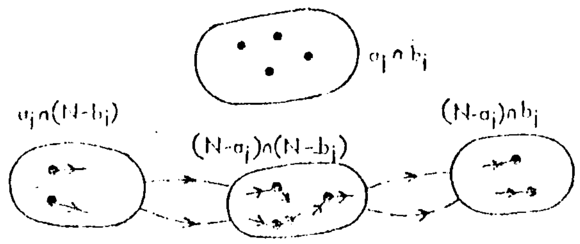


Fig. 2. Necessary incidence in the remaining graph.

The above interpretation gives us an important corollary.

Corollary 2. If we want to satisfy conditions /7/ then after deleting all the edges outgoing from b_j vertices we should find such a set a_j that after deleting all the edges incoming to a_j vertices there are no isolated vertices in the set $N - (a_j \cap b_j)$.

Nonzero rows and columns of $Y_n(a_j | b_j)$ do not guarante that this matrix is nonsingular. However, it may be singular for particular values of elements only. If we know designed nominal values of the elements then we can easily check wheather the tests a_j chosen are sufficient for the solution. If no, we change tests.

Definition 1. A vertex of a complete subgraph of the flow graph is said to be a corner if there are no edges incoming to the vertex from out of the subgraph.

It follows from the above definition that there may exist edges outgoing from a corner to other parts of the graph. Also, the order of the complete subgraph is not defined - in particular it may be a complete graph of zero order /see Fig. 3a/

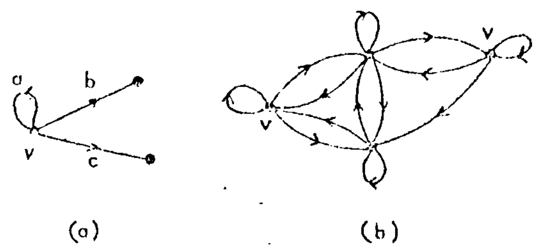


Fig. 3. Examples of corners. Corners are denoted by v.

Based on corollary 2 the following theorem can be proved.

Theorem 2. Regardless of the choice of a sequence of b_j, $j=j_1, \ldots, j_N$, the indices of all corners have to appear in the set a.

Thus, the number of corners estimates minimal cardinality of a. In order to further estimate this the following remarks may be useful.

Remark 1 card(a) $\geqslant$ maximal order of a complete subgraph

Remark 2 card(a) $\geqslant$ minimal incoming degree in the remaining graph after deleting all corners with incident edges,

where incoming degree of a vertex is the number of edges incoming to this vertex.

3. ALGORITHM

We now describe a strategy of the choice of a sequence of b_j, representing so-called reduced cut sets. The method is somehow heuristic, so the algorithm does not pretend to provide the best possible solution. However, our goal is to find a set of tests of a reasonably small cardinality.

The flow graph may have such a structure that there exist subgraphs, obtained by a sequence of bisections, that the flow between them is in one direction only. We call such subgraphs as weakly connected subgraphs. An example is shown in Fig. 4.

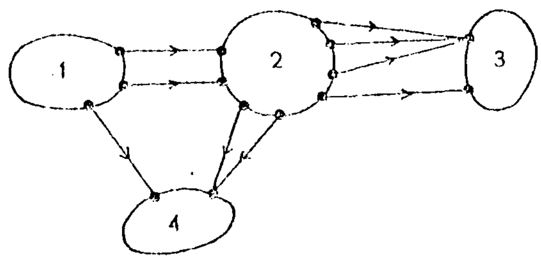


Fig. 4. Partition of a graph on weakly connected subgraphs.

The following steps set the algorithm out in sufficient detail.

Algorithm

Data — Coates graph of the network considered.

Step 1 — Partition the graph into weakly connected subgraphs. Set M ← number of weakly connected subgraphs. Number the subgraphs in such a way that the subgraph without outgoing edges to other subgraphs is chosen as the first one, then remove this subgraph and repeat the procedure.

Comment — For a passive network M=1. Renumbering of subgraphs of Fig. 4 will be 1 ← 3, 2 ← 4, 3 ← 2 and 4 ← 1.

Step 2 — Set LS ← 1, P(i) ← 0 for i=1, . . . , M and a ← ∅.

Comment — LS denotes the index of a weakly connected subgraph and P(i) denotes the minimal cardinality of a for the i^{th} subgraph based on the information available.

Step 3 Stop if LS > M.

Step 4 Find corners of the LS^{th} subgraph and include their indices in the set a(LS). Set P(LS) ← max(P(LS), number of corners).

Step 5 Delete all the edges incoming to the corners and all the edges with the same weights. Indices of the vertices incident with the edges incoming to the corners form first sets b_j j=1, ..., P(LS). Include the vertices, except the corners, such that the edges deleted were directed towards them to the set of active vertices AV.

Step 6 In the remaining subgraph LS find the vertex v with the minimal non-zero incoming degree /see Remark 3/. If there is no such vertex, augment the set a(LS) using latest vertices of AV to satisfy the condition

$$card(a(LS)) \geq P(LS)$$

and go to step 10.

Step 7 In the set AV find the vertex va with the minimal incoming degree d'(va). Set x ← va

Step 8 If d'(va) > max(d'(v), P(LS), card(a(LS)) +1) then include v in a(LS) and set P(LS) ← max(P(LS), d'(v)), x ← v Otherwise, set P(LS) ← max(P(LS), d'(va)) and exclude va from AV.

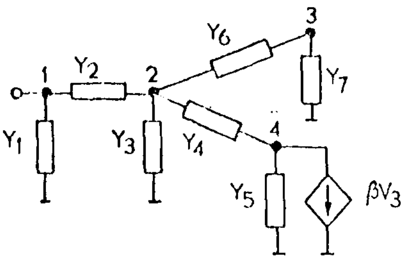
Step 9 Form the subsequent set b_j by taking indices of the vertices incident with the edges incoming to x. Delete all the edges incoming to x and all the edges with the same weights. Include to AV such vertices, except x, that the edges deleted were directed towards them. Go to Step 6.

Step 10 Set a ← a ∪ a(LS), LS ← LS+1 and assign P(LS) according to Remark 3. Go to Step 3.

Remark 3 When calculating the incoming degree of v or va we do not count the edges outgoing from another subgraph i. The maximal number of such edges incoming to a vertex x which apear in b_x determines initial value of P(i).

4. EXAMPLE

As an example we apply the algorithm proposed to the active circuit shown in Fig. 5 /a/. The corresponding Coates graph is drawn in Fig. 5/b/. For this circuit M=1 and we find two corners: 1 and 3. Using the algorithm we generate the sequence of b_j as {1, 2}, {2, 3}, {2, 4}, {3, 4} for which the set a={1, 3} does not need to be augmented, so the tests 1 and 3 are sufficient for the identification of the elements $Y_1, \ldots, Y_7, \beta$.



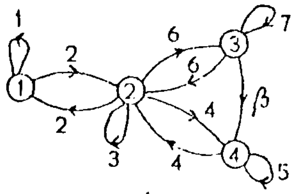


Fig. 5/a/ An active circuit example, /b/ its Coates graph.

It is worth mentioning that for a similar circuit, but without the current source, three tests 1, 3 and 4 are required.

5. CONCLUSIONS

The algorithm presented enables us to find a reasonable small number of tests which are topologically sufficient for identification of all component values of linear analog circuits. This has been achieved due to searching for a "good" sequence of reduced cut-sets, whose elements are consecutively determined from Eq. /6/. The notion of a corner is particularly important, since it determines necessary tests independently of a sequence of cut-sets. Obviously, if the corners demonstrate sufficiency then our algorithm provides the best possible solution. The method is easy to program and gives a linear dependence of computational effort on the size of the network.

6. REFERENCES

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