A New Method to Compensate Quantized Time-Base Nonlinearity of Sampling Instruments

Dong Liu*, Janusz A. Starzyk*, Gerard N. Stenbakken**, and Bryan C. Waltrip**

*School of Electrical Engineering and Computer Science, Ohio University, Athens, OH 45701 ** National Institute of Standards and Technology, Gaithersburg, MD 20878

Presenting person: Dong Liu, Phone (740) 593-4553, Email: dliu@bobcat.ent.ohiou.edu

Abstract---Timebase distortions cause nonlinear distortions of the sampled waveforms by equivalent-time sampling instruments. A new model of quantizer is proposed to be used when a large part of timebase distortions is the result of non-random quantization error that is known. A compensation method based on this model is applied to timebase quantization. The simulated results show that the proposed compensation method can reduce the cumulative timebase quantization error, change the non-normal rms quantization error to be normal, and improve the measurement accuracy. The experimental results from a wideband sampling voltmeter confirm that the proposed method can improve the measurement accuracy of sampling instrument.

Index Terms---distortion error, error modeling, error simulation, nonlinear timebase, quantization error.

I. INTRODUCTION

Nominally uniformly spaced equivalent-time sample intervals are fundamental for the modern sampling instruments based on equivalent-time sampling principles. The deviations away from such time intervals of equivalent-time sampling instruments have two components: a random part called jitter or aperture uncertainty, which is not the subject of this paper, and a deterministic part called time-base distortion which is the main topic discussed in this paper. Timebase distortions cause nonlinear distortions of the sampled waveforms. If the timebase distortion is caused by non-random quantization, the sampling intervals can be modified, so that the values of the sampled signal at the nominal sample times can be compensated and measurement accuracy can be improved.

A. Previous Work

Among the efforts of previous research on quantization, Widrow's quantization theorem [1] is the standard analytical tool in use, which is based on statistics of the random process. Uniform random model of quantization error is traditional and fundamental [1-4]. Fig. 1 illustrates the uniform model of quantization error widely adopted to describe the quantization error: the output x' of the quantizer is modeled as a sum of an infinite-precision input x and an independent random quantization error n. This quantization error is uniformly distributed, and has a mean of zero and a mean square of $q^2/12$, where q is the quantization step. This uniform random model is usually applied to amplitude quantization of the sampled waveform.

For the uniform model, quantization error is random and independent of the input signal, but quantization process is not always random. Under some conditions, quantization process is non-random: quantization is determined by the signal parameter, thus quantization error is known and quantization is not random. For example, quantization is dependent upon the input signal frequency, resolution of quantizer, start phase of input signal, etc..

B. Scope of this Work

This paper describes a new model of quantizer which is applied to timebase quantization to compensate timebase distortion. The new model makes use of the known quantization error to describe the measurement effects of this timebase nonlinearity component. A proposed compensation method can reduce the cumulative timebase quantization error, change the non-normal rms quantization error to be normal, and improve the measurement accuracy of sampling instruments.

II. SIMULATION SETUP

A. Simulated System

The system simulated here is an equivalent-time wideband sampling voltmeter (WSVM) developed at the U.S. National Institute of Standards and Technology (NIST) [5]. A generator provides sine waveform with the programmable amplitude, phase and frequency as the input signal of the WSVM. Synchronous trigger signal is also provided as the external trigger source of the WSVM.

The WSVM is a high-accuracy sampling voltmeter for rms measurements of repetitive signals ranging in frequency from 10 Hz--200 MHz. A custom strobed comparator is applied as the sampling device and decision element. The timebase design is based on the classic ramp or sweep generator technique in which a trigger signal initiates a linear voltage ramp. Sample commands are provided by timebase circuit for each repetition of the ramp. Sample commands for the comparator are generated when the reference level (generated by timebase DAC) is crossing the ramp voltage. The WSVM utilizes equivalent-time sampling technique for the input signal frequency above 50 kHz, and utilizes a novel quasiequivalent-time sampling method for the frequency below 50 kHz. Fig. 2.a illustrates the equivalent-time sampling principle: the input signal is resampled at the same equivalent internal before each step of the quantization process, by means of the strobed comparator. Fig. 2.b illustrates the quasiequivalent-time sampling principle: multiple samples are performed per repetition of the input signal, thus can reduce the sampling times. Here is shown an example of eight sample points per period of the waveform that use two timing ramps to complete the desired number of samples.

As indicated above, the reference level is provided by a precision DAC in the timebase circuit, which is calculated in advance to give the desired time delays. The quantization can be known from the DAC level, thus is dependent upon the input signal parameters. Non-random quantization is applied to this case.

B. The modified Quantizer

Traditionally for random quantization, input signal x is directly input to the quantizer and output x' is then obtained and quantization error n, which is defined by n = x'-x, is discarded [Fig. 1.a]. A new model of quantizer is proposed in Fig. 3.a for non-random quantization in which the quantization error n is known. A new compensation method that makes use of new model is presented in Fig. 3.b: before feeding to the quantizer, the input signal x is added with the sum of previous quantization errors $\sum n \cdot \sum n$ is defined

as $\sum n = 0$ for i = 1, and $\sum n = \sum_{i=1}^{i-1} (x'_i - x_i)$ for $i \ge 2$, where *i* is the index number of

sampling length N, $1 \le i \le N$. This compensation method is applied to timebase quantization.

We refer to the error from the new organization of the quantizer as the modified quantization error while the error from the quantizer shown in Fig. 1 is referred to as the normal quantization error.

III. RESULTS

A. Simulation Results

First, the effect of applying this model to timebase quantization is the reduction of the cumulative timebase quantization error. Figs. 4 are the simulation results: 128 samples are taken within one period of the input sine signal with frequency of 16 kHz, the quantizer is a 10-Bit DAC. Fig. 4a illustrates the time-base quantization error by the normal method, with the quantization error ranging between -0.5 and 0.5 of the least significant bit (LSB) of the quantizer. Fig. 4b shows the corresponding cumulative time-base quantization error. The cumulative error ranges between 0 and 8 LSB, which is biased and is much greater in quantity than single time-base quantization error showing in Fig. 4a. Fig. 4c displays the time-base quantization error obtained by the modified method, with the quantization error ranging between -1 and 1 LSB of the quantizer. Fig. 4d demonstrates the corresponding cumulative time-base quantization error. The range is between -0.5 and 0.5 LSB, which is unbiased and much smaller in quantity than the results obtained by the normal method in Fig. 4b.

With the proposed method, the non-normal distributed rms quantization error can be changed to be normal, and accuracy of the rms quantization error by this method is also improved. Simulations are performed over the frequency range of 10 Hz-200 MHz with a 10-Bit DAC. In Figs. 5, the horizontal axis stands for the simulated rms quantization error, while the vertical axis stands for the rms quantization error predicted to be normally distributed. If the simulated results are normally distributed, a straight line will appear. But in Fig. 5.a, there is only a limited section of the curve which has a linear pattern around vertical axis, while long "tails" at each end of the curve appear. It can be concluded that the rms quantization error of the normal method is not normally distributed, and the normal quantization process is non-random. Fig. 5.b is the corresponding plot using the modified model. It approximates very well a normal distribution, so the rms quantization error is changed to be normal by the modified model.

The measurement accuracy is also improved from the level of 10^{-3} in Fig. 5.a to the level of 10^{-5} in Fig. 5.b. The modified method can improve measurement accuracy greatly.

B. Experimental Results

Measurements from the WSVM were performed with the normal and the modified methods and the measured results matched the simulated results.

Fig. 6.a and 6.b show the results from the measurements: 10-Bit DAC is utilized and 200 samples were taken over the frequency between 52 kHz and 52.8 kHz, in which

equivalent-time technique is utilized by the WSVM. Fig. 6.a is obtained with the normal method while Fig. 6.b is obtained with the modified method. The range of rms quantization error in Fig. 6.b is between $-3 \times e^{-4}$ and $3 \times e^{-4}$, which is about the 1/6 of the range in Fig. 6.a, $-1.8 \times e^{-3}$ and $1.8 \times e^{-3}$. It indicates that the modified method can reduce the rms quantization error as predicted by simulation.

The amplitude range of the simulated rms quantization error with the normal method is between $-2 \times e^{-3}$ and $2 \times e^{-3}$ [Fig. 5.a]. The corresponding measured results are on the same amplitude level [Fig. 6.a]. The amplitude range of the simulated rms quantization error is between $-8 \times e^{-5}$ and $9 \times e^{-5}$ for the modified method [Fig. 5.b]. But the corresponding measured results are between $-3 \times e^{-4}$ and $3 \times e^{-4}$ [Fig. 6.b]. This is caused by electronic noise of the WSVM: noise level of the WSVM is $4 \times e^{-4}$ [5], which is on the same amplitude level as the results in Fig. 6.b. The measured results with the modified method have the same amplitude level as the noise level of the WSVM because noise level of the WSVM overrides the resolution provided by the modified method. The conclusion can be drawn that the amplitude levels of the measured results match the amplitude levels of the simulated results.

IV. CONCLUSIONS

A new method is presented to model the non-random timebase quantization nonlinearity of sampling instruments. A compensation method is used to modify timebase quantization of equivalent-time sampling instruments. With this modified method, the cumulative time-base quantization error is reduced. The non-normal distributed rms quantization error is changed to be normal. The measurement accuracy of sampling instruments is also improved. The experimental results from a wideband sampling voltmeter developed at NIST confirm that this proposed method can improve the measurement accuracy of the sampling voltmeter.

REFERENCES

[1] B. Widow et al., "Statistical Theory of Quantization," IEEE Trans. Instrum. Meas., vol. 45, no. 2, pp. 353-361, Apr. 1996.

- [2] M. F. Wagdy and Wai-Man NG, "Validity of Uniform Quantization Error Model for Sinusoidal Signals Without and With Dither," IEEE Trans. Instrum. Meas., vol. 38, no. 3, pp. 718-722, Jun. 1989.
- [3] H. B. Kushner et al., "Almost Uniformity of Quantization Errors," IEEE Trans. Instrum. Meas., vol. 40, no. 4, pp. 682-687 Aug. 1991.

[4] K. Hejn and A. Pacut., "Generalized Model of the Quantization Error-A Unified Approach," IEEE Trans. Instrum. Meas., vol. 45, no. 1, pp. 41-44, Feb. 1996.

[5] T. Michael Souders et al., "A Wideband Sampling Voltmeter", IEEE Trans. Instrum. Meas., vol. 46, no. 4, pp. 947-953, Aug. 1997.



Fig. 1. Uniform model of quantizer (copied from [1]):(a) quantization error addition;(b) PDF of quantization error.



Fig. 2.b Quasiequivalent-time sampling (copied from [5])



Fig. 2.a Equivalent-time sampling (copied from [5])



Fig. 3 Modified quantizer: (a) new model of quantizer; (b) compensation method.



Fig. 4 (a) Timebase quantization error before modification; (b) Cumulative timebase quantization error before modification; (c) Timebase quantization error after modification; (d) Cumulative timebase quantization error after modification.



Fig. 5 Simulation results over frequency of 10 Hz-200 MHz with 10-Bit DAC: RMS quantization error predicted to be normally distributed versus simulated RMS quantization error (a) before modification; (b) after modification.



Fig. 6 Measured results from WBSVM over frequency of 52 kHz-52.8 kHz with 10-Bit DAC: measured RMS quantization error versus frequency. (a) before modification; (b) after modification.