A software program for ambiguity group determination in low testability analog circuits

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Abstract

A software program that can be considered as a tool for the fault diagnosis of analog linear circuit is presented. The proposed program implements a numerically efficient approach to identify complex ambiguity groups in the case of circuit of great size and low testability. This approach is based on an efficient QR factorization technique applied to the testability matrix. An amplifier circuit is shown as example of application of the software program.

1 Introduction

In analog circuit testing, an effort of fundamental importance is constituted by fault diagnosis and fault location. These testing phases play an essential role in design validation and prototype characterization because they allow to improve the yield through design modification. In the past years, fault diagnosis and fault location received the attention of many researchers (see, for example, [1-5]), especially for what concerns the automation of fault diagnosis procedures [6]. In fact, for analog and mixed digital-analog systems, the lack of simple fault models and the presence of component tolerances and circuit nonlinearities make the automation of fault diagnosis and fault location procedures very complex. As a consequence, while for digital circuits fully automated fault diagnosis techniques are commonly used, for analog circuits, the development level is less advanced. The information obtained by the ambiguity groups determination helps to find unique solution of fault diagnosis equations or identifies which groups of components can be uniquely determined.

When, as it usually happens, the testability value is less than the total number of potentially faulty circuit components, the problem is not uniquely solvable and it is necessary to consider further measurements, or to accept a reduced number of potentially faulty components for the circuit under consideration. Since not all test points are feasible due to the practical and economic measurement constraints, and since the number of faulty components is generally smaller than the total number of circuit components, the fault diagnosis problem must be curtailed. In such situation a quite realistic hypothesis is assumed that the number of faulty components is bounded, that is the k-fault hypothesis is made.

Under this hypothesis, in order to locate the faulty elements with as low as possible ambiguity, it is of fundamental importance to determine a set of components which is representative of all the circuit elements. To accomplish this, it is necessary to determine not only the circuit testability but also the canonical ambiguity groups. Roughly speaking, an ambiguity group is a set of components that, if considered as potentially faulty, does not give a unique solution in fault location. A canonical ambiguity group is simply an ambiguity group that does not contain other ambiguity groups.

An efficient algorithm for the computation of ambiguity groups has been proposed by Stenbakken, Souders and Stewart [7]. Recently the authors [8] have developed a new method for the determination of all the canonical ambiguity groups of a circuit. This algorithm finds all possible ambiguity groups and all the sets of circuit parameter values that are consistent with the test equations. However, the proposed algorithm is combinatorial in nature and is useful only for small analog circuits. A new method to overcome this limitation uses the QR factorization of the circuit testability matrix [9].

In this paper a software program implementing the algorithms proposed in [9] for determining all the possible ambiguity groups of a linear analog circuit with low testability and all the sets of circuit parameter values which are consistent with the test equations is presented. The program is based on an efficient QR factorization of the circuit testability matrix and on the use of symbolic analysis techniques. The symbolic analysis is a procedure that permits one to obtain, as a result of a computer

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program, circuit network functions in a closed form, where some or all the circuit elements and the complex frequency are represented by symbolic parameters. Hence, the symbolic analysis is particularly suitable for testability evaluation and ambiguity group determination, where the component values are the unknown quantities. It allows to realize very simple algorithms dedicated to these problems by exploiting the determination of the nonlinear fault diagnosis equations in symbolic form with respect to both complex frequency s and component values [10] and, subsequently, the sensitivity analysis of these equations [11].

2 Theoretical basis and software organization

For low testability system, no simple solution can be found using traditional solving methods for system test equations, because they are singular. In what follows we concentrate on this kind of systems. If the fault equations are known, the potentially faulty parameters vector P (the faulty parameters number is equal to p) can be related to the measurements vector V (the number of measurements is m) by using the testability matrix B as follows:

$$B\,P = V \tag{1}$$

where B is testability matrix of order mxp, obtained by the Jacobian of the fault equations or it is a matrix used in linear verification techniques [12-13]. Vectors P and V constitute the parameters variations and measurement variations with respect to the nominal circuit, if the testability matrix is obtained from the Jacobian. If the test equations are derived as discussed in [12-13], the two matrices directly represent the parameter values and measurements, respectively. In both cases, each circuit parameter is related to a specific column of the testability matrix. Hence the ambiguity groups determination is based on the correct identification of the sets of linearly dependent columns that belong to the testability matrix. Let us remind that we are considering low testability circuits, hence the matrix B does not have the full column rank. In such case, the matrix B can be partitioned into two submatrices B=[$B1$ $B2$] which are linearly dependent:

$$B2 = B1\,C \tag{2}$$

where mxr matrix $B1$ has the full column rank equal to the rank of the matrix B, and the columns of rx(p-r) matrix C represent an expansion of the corresponding columns of $B2$ in the basis vectors obtained from the columns of B1 . Due to the role of the matrix C in the expansion of the dependent columns of B into a set of the basis columns we call C a linear combination matrix. Moreover when the QR factorization is performed the following equation can be written:

$$B\,E = Q\,R \tag{3}$$

where E is pxp column selection matrix, Q is a mxm orthogonal matrix and R is mxp upper triangular matrix. In the presence of ambiguity groups in the testability matrix B, its rank and the rank of R are less than p. Therefore, matrix R can be written as

$$R = \begin{bmatrix} R1 & R2 \\ 0 & 0 \end{bmatrix} \tag{4}$$

where $R1$ is rxr upper triangular and has its rank equal to the rank r of the testability matrix B. Hence the linear combination matrix C can be numerically obtained from the QR factorization of the testability matrix B using:

$$C = R1^{-1}\,R2 \tag{5}$$

Furthermore the product $B\ E$ = [$B1$ $B2$] constitutes a partition of B that defines C, with the matrix $B1$ that corresponds to $R1$ representing the independent set of columns. From the definition of the matrix it can be noticed that different partitions define different linear combination matrices C.

We have also introduced the definition of basis of a partition as the set of components that correspond to columns of matrix B1 and the co-basis as a set of components that correspond to columns of matrix $B2$. In [9] it has been demonstrated that is possible to determine a combination matrix C written in a minimum form, that is a matrix with a maximum number of zero elements. A minimum form C is not unique. It is enough to switch a component of the basis (that corresponds to a row of C with a single nonzero component) with the corresponding component of the co-basis (that corresponds to a column which includes this nonzero component) to obtain another minimum form of C. The corresponding partition (2) is called a canonical form of the testability matrix [9].

Moreover, as is well known, the testability measure defined as the rank of the testability matrix is independent on parameter values, which means that the rank of the testability matrix is equal to a given testability measure almost everywhere in the parameter space. This result can be extended to ranks of all submatrices of the testability matrix that are used to determine the existence of ambiguity groups. Under this assumption we may study properties of the linear combination matrix C considering its equivalent binary matrix D that has the same size as C

and it can be obtained by considering equal to one the corresponding elements of *C* that are different from zero while the remaining ones are considered equal to zero. As in matrix *C*, rows of *D* correspond to the elements of the basis and columns correspond to the elements of the co-basis on a given partition of the testability matrix. This equivalent representation simplifies the analysis of C as the set theory can be used to study its structural properties.

It has been shown in [9] that a suitable elaboration of the matrix D allows us to determine all the canonical ambiguity groups, hence the clusters of ambiguity and the surely testable components of the circuit under test without considering all the possible combinations of sets of columns and in a shorter number of iterations. In fact the computational complexity of this approach that identifies ambiguity groups is on the order of O(p3) that is much smaller than the computational cost of checking all combinations of p columns that is on the order of O(2^p p^3). This exponential dependence of the search time on the number of tested parameters renders ambiguity analysis by using the previous combinatorial approach, impractical for all but very small designs. Hence this new procedure has been implemented in a software program that, starting from the circuital scheme of the device under test, is able to determine in an efficient way the ambiguities information also in the low testability case.

A program called ATES (ambiguous test equations solver) was developed using Matlab. Using B and M matrices as input, it automatically gets all useful results quickly, finding ambiguity groups, minimum form of matrix D, and the solution of faulty parameters. Fig. 1 illustrates the software organization of program ATES.

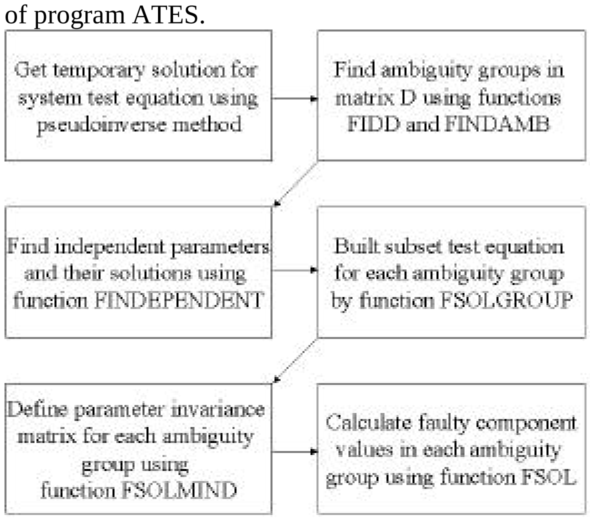
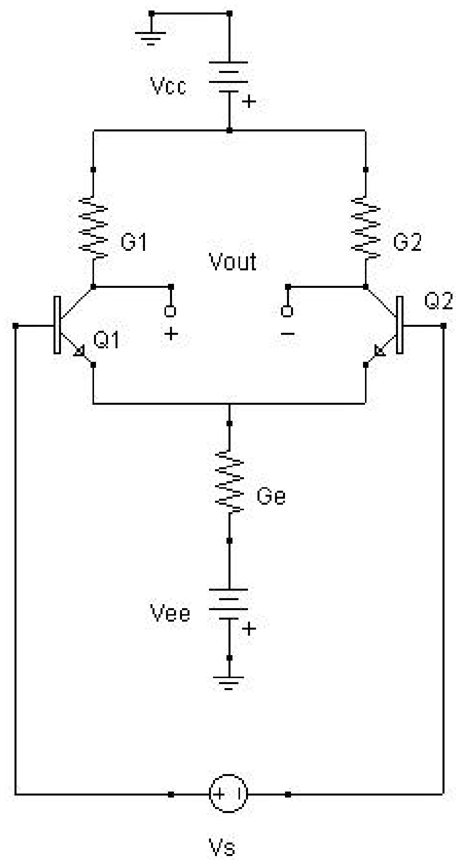


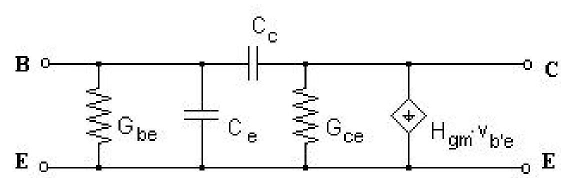
Fig.1. *Software Organization of ATES*

3. Circuit example

Let us consider an IC differential amplifier as an example to which the procedure has been applied. Fig. 2 shows the amplifier's topology and the π transistor model used in simulation.



a) IC differential amplifier



b) Transistor model

Fig. 2 *Differential amplifier as a low testability circuit*.

The program Sapwin [14] has been used and integrated with the new software to produce the fault equation, written in a symbolic form, corresponding to the voltage answer to a differential voltage input. Using such test point, the testability value is equal to six while the circuit parameter number is equal to thirteen: we are in a low testability case. Hence the herein presented procedure has been applied to the fault equation giving the results of Fig. 3

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Circuit Testability: 6

Total number of components: 13

Canonical Ambiguity groups:

Ge   G1   G2   Gce_Q1
Ge   G1   G2   Gce_Q2
Ge   G1   Gce_Q1   Gce_Q2
Ge   G2   Gce_Q1   Gce_Q2
G1   G2   Gce_Q1   Gce_Q2

Ambiguity clusters:

Ge   G1   G2   Gce_Q1   Gce_Q2

Surely Testable components:

Gbe_Q1   Ce_Q1   Hgm_Q1   Gbe_Q2   Ce_Q2   Hgm_Q2   CC_Q1   CC_Q2
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Fig. 3 *Software program results*.

It is important to notice how information about the circuit ambiguities constitutes a fundamental step of the fault diagnosis process. In fact in this case the minimum size of the canonical ambiguity groups is equal to four, allowing a unique identification for the potentially faulty parameters for the case of double fault [8], whatever fault location procedure will be used following the fault diagnosis process.

4 Conclusions

A software program for the determination of ambiguity groups in low testability systems has been presented. The developed algorithm has a very modest computational cost comparing to the combinatorial method proposed in [8]. This allows to efficiently deal with analog circuits that are more complex and of a larger size than those considered in [8]. Although a unique solution is not always possible in such systems, our method provides the best possible alternative. The method provides a unique solution for all testable components. In addition, components within an ambiguity group have unique solution under the assumption of the number of faults being smaller than the rank of the corresponding ambiguity group. The program results can constitute the first step in the development of a procedure for the fault location of analog linear circuits, because they represent theoretical and rigorous upper limits to the degree of solvability of the faulty component location in low testability circuits.

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