Robust Optical Flow Estimation Using Invariant Feature

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Abstract – Traditional methods for computing optical flow are mainly based on image brightness constancy. In the real world the brightness constancy usually does not hold. Here we present the idea of using invariant feature based on the brightness change model to estimate the optical flow. Both the mathematical derivation and the experiments show that the new model is better than brightness based optical flow constraint.

1. *INTRODUCTION*

Optical flow as defined by Horn [1] is the apparent motion of brightness patterns observed when a camera is moving relative to the objects being imaged. Gradient-based methods are the most widely used methods to compute the optical flow. Although almost all gradient-based methods such as the method proposed by Horn and Schunck's [1] assume the image brightness doesn't change as the object changes its position in space, it is not proper to assume the invariance of recorded image brightness along motion trajectories on which the gradient equation is based.

Several authors have discovered the problem and tried to model the optical flow using more accurate models. Schunck [2] proposed the extended optical flow constraint (EOFC) to overcome the limitations of the basic optical flow constraint OFC), but he did not provide the theoretical background of his EOFC. Bimbo et.al [3] compared the EOFC and OFC both in theory and in experiment, and he discovered that the EOFC was not a better model of optical flow than OFC. Negahdaripor [4] proposed the general dynamic model of image motion (GDIM) which can accommodate the change of brightness. The model was more general, but with the two new parameters it is more difficult to estimate the optical flow in the later process.

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Other researchers focused on the mathematical expression of the brightness change. Nagel [5] derived analytical expressions for image irradiant variations due to perspective projection effects for a moving Lambertian surface patch under isotropic illumination. A generalization of the basic gradient constraint was also obtained by introducing more sophisticated photometry models of image formation (Verri and Poggio, 1989) [6]. However, these models cannot be easily identified and generally do not lead to practical implementation.

Ghosal et.al [7] took a practical view of how to improve the accuracy of estimating the optical flow. Instead of studying the brightness change model mathematically, they use the Zernike moment to replace the brightness and hope to use the invariant property of moments so that although the brightness may change a lot the moment changes a little. They did not provide any adjustment in theory except for experiment results

In this paper, we present a simple method to improve the constancy of optical flow constraint. The model we take makes use of the Nagel's derivation of brightness change. Both the mathematical derivation and the experiments show the new model is better than brightness based OFC.

In section 2 we discuss some backgrounds of the optical flow constraints; in section 3 the general idea of improving the optical flow constraints is presented, and section 4 deals with a special realization of the general idea. The experiments are discussed in section 5.

II. BACKGROUND

Some basic notions should be stated. Given a point P in 3-D space identified by the vector $\vec{P} = (X, Y, Z)^t$, its perspective projection on the image plane with focal length Z=f is $\vec{p} = (x, y)^t$. The following relationship holds:

$$\vec{p} = \frac{f}{Z}\vec{P} \qquad (1)$$

The projection of the 3-D motion on the image plane refereed to as 'velocity field' is obtained by taking derivatives on (1):

$$\dot{\vec{p}} = \frac{f}{Z}(\dot{\vec{P}} - \vec{P}\frac{\dot{\vec{P}} \cdot \hat{Z}}{Z}) \qquad (2)$$

where $\hat{Z}$ is the Z-axis unit vector. The 3-D motion of the generic point P can be modeled as comprised of transnational and rotational velocity components:

$$\dot{\vec{P}} = \vec{W} + \vec{\Omega} \times \vec{P} \qquad (3)$$

where $\vec{W} = (W_1 £ \ W_2 £ \ W_3)^t$ and $\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^t$ are the instaneous translation and rotation respectively. Submitting(1)and(3) in(2), two scalar equations are obtained which express relationships between the components u,v of velocity field $\dot{\vec{p}}$:

$$\begin{cases} u = \frac{fW_1}{Z} - x\frac{W_3}{Z} + f\Omega_2 - y\Omega_3 - x\frac{y\Omega_1 - x\Omega_2}{f} \\ v = \frac{fW_2}{Z} - y\frac{W_3}{Z} + x\Omega_3 - f\Omega_1 - y\frac{y\Omega_1 - x\Omega_2}{f} \end{cases} \qquad (4)$$

Here we refer to u and v as velocity field. The optical flow is defined as the solution of the optical flow constraints. Studying the accuracy of the optical flow model is to study how optical flow constraints can be used to approximate the velocity field. For further information of optical flow field and velocity field, turn to reference [3].

Due to the assumption on constancy of the image brightness, $dE/dt = 0$, Horn and Schunck [1] get their basic optical equation:

(5)

Schunk [2] felt that the original motion constraint equation would hold neither for rotational motion nor for images created by nonparallel projection. Schunk argued that the compression or expansion of the edge fragments due to this foreshortening would be modeled by the continuing equation for the dynamics of compressible fluids. Based on ideas taken from fluid dynamics and transport theory, he derived the following constraint equation (EOFC):

$$E_x u + E_y v + E u_x + E v_y + E_t = 0 \qquad (6)$$

Nagel felt that a constraint equation for the estimation of optical flow should be based more explicitly upon the geometric properties of the 3-D scene and perspective projections. In his paper, he derived a new constraint equation based on a combination of perspective projection and notions from differential geometry.

Nagel's 3-D parameters optical flow equation is expressed as:

(7)

However good the theoretical result is, all the 3-D parameters in this equation are unknown, which makes the equation useless in practice.

III. IMPROVING THE ACCURACY OF OPTICAL FLOW CONSTRAINT

The general idea of improving the accuracy of the optical flow constraint is to use the feature other than the brightness. The new feature should be more stable than brightness, and it should also reflect the brightness change model such as the 3D model discussed in Nagel's paper [5]. Besides the new feature forming the generalized optical flow constraint should also have the ability to preserve the normal flow defined by the brightness constancy model.

In mathematical representation, we would like to define a local transform for every pixel:

$$M_{ij} = f(\Omega_{ij}) \qquad (8)$$

where Ω_{ij} is the neighborhood around the pixel location (i,j).

(8) defines a feature based on the brightness of the local neighborhood, and it should have the following property.

Property 1: $\frac{dM}{M} \le \frac{dE}{E}$ (9)

Property 2: $$\left\| \frac{M_t}{\sqrt{M_x^2 + M_y^2}} - \frac{E_t}{\sqrt{E_x^2 + E_y^2}} \right\| \to 0 \quad (10)$$

Property 3: Defining M should take consideration of brightness change model, such as (7).

Property 1 tells us the new feature should be more invariant than brightness. Property 2 requires the preservation of the normal flow. If we only have property 1, some trivial feature may be chosen, such as mapping all values to constant, which is obviously not a desired solution. Property 2 constrains the choice of the transformation, and it also tells us if the brightness constancy is hold at that point, using feature M to estimate the optical flow should give the similar result of using brightness constancy model. Property 3 is needed to further confine the choice of local transformation, the feature should encode the information provided by the brightness change model. The idea is quite clear, but it is very difficult to form it as mere mathematical optimization problem.

IV. OPTICAL FLOW BASED ON INVARIANT FEATURE

As discussed in section 3, the improvements of optical flow constraints rely on some sort of local transformations, which provide more robust feature than the brightness. Here we present one kind of such kind of features and discuss how it conforms to the three properties.

The feature we are using is defined as :

$$M_{ij}(t) = \frac{E_{ij}(t)}{\sum_{st \in \Omega} E_{st}(t)} \quad (11)$$

where E is the brightness of the image point, i,j,s,t are the indexes, Ω is the neighborhood around i,j.

Let's discuss how the feature M conforms to three properties.

Lemma 1: The formula (7) is equivalent to

$$\left(E_x + 4E\frac{x}{x^2+y^2+f^2} \right)u + \left(E_y + 4E\frac{y}{x^2+y^2+f^2} \right)v + E_t = 0 \quad (12)$$

The proof is merely algebra calculations using the perspective camera model (1) and three-dimensional motion model (3), we omit it here.

Lemma 2: For every pixel $\frac{dM}{M} \le \frac{dE}{E}$

The proof is based on (12). From equation (12) we can get the brightness change which is expressed as

$$-4E\left(\frac{x}{x^2+y^2+f^2} + \frac{y}{x^2+y^2+f^2} \right) \quad (13)$$

with some algebra calculation we can easily prove Lemma 2.

Considering a special case, if the size of object patch is much smaller than its distance to the camera, dM will approaches 0. This can be easily verified using (13) and (11).

Lemma 1 can also be used to study the usage of moments as invariant feature to improve the OFC such as the feature presented in [7]. It can be proven that using the moments is neither better than using brightness nor worse than it. They are almost identical in the sense of property 1. Why in paper [7] they reported the better experiment results? It is because in the real world, there is noise in the image, and the low order moments especially Zernike moments are very robust to noise.

This leads to the following further improvement of our method. After we getting the feature M, we calculate the Zernike moment of every pixel based on M.

A robust optical flow estimation technique is proposed here based on the principle of conservation of a set of local M functions, characterized by locally computed Zernike moments. Zernike moments are orthogonal complex moments and are projections of image data onto a set of orthogonal polynomials within a unit circle. Here we replace the image data with our M function, and the moment of order n and repetition m can be defined as:

$$A_{nm} = \iint_{x^2+y^2 \le 1} M(x,y) V_{nm}^*(x,y)\,dx\,dy \quad (14)$$

where $V_{nm}^*(x,y)$ is the moment generating polynomial

and can be expressed as $V_{nm}^*(x,y) = R_{nm}(\rho)e^{jm\theta}$

where

$$R_{nm}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s (n-s)!\,\rho^{n-2s}}{s!\left(\frac{1}{2}(n+|m|)-s\right)!\left(\frac{1}{2}(n-|m|)-s\right)!}$$

Therefore, given an invariant moment based feature

$A(x,y,t)$ at an image point (x,y) we get like Horn and Schunck

$$\boxed{\qquad\qquad\qquad\qquad} \qquad (15)$$

The following procedure seems to be clear. We will implement the Lucas and Kanade's method using (15) instead of (5).

The method has several advantages: first from brightness E to feature M we get better resistance to brightness change and from feature M to feature A we get better noise resistance. Second, since we can use different order and repetition of the moment, it is easy to form a set of over-determined equations to estimate the optical flow.

Considering property 2, we should not use the feature A everywhere. The following condition is considered: if the current point is an edge point, we use brightness constancy model to calculate the optical flow, in other cases we use feature A.

V. EXPERIMENT

The experiment is based on the synthetic images. The synthetic image sequence is as follows: The image is perspective projects of the 3-D sphere, the equation of the sphere is $X^2 + Y^2 + (Z-1000)^2 = 50^2$, and the albedo of the sphere surface is [3]

$$r = 100\sin\left(\frac{2\pi X}{10}+\frac{\pi}{2}\right)\sin\left(\frac{2\pi Y}{10}+\frac{\pi}{2}\right) \bullet \sin\left(\frac{2\pi(Z-950)}{10}+\frac{\pi}{2}\right)+155 \qquad (16)$$

the light source is isotropic, the inner parameters of the camera set as:

$$(f, D, a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}) = (1, 1, 0, 1/1000, -63/1000, -1/1000, 0, 63/1000) \qquad (17)$$

The method to solve the equation is identical to Lucas and Kanade's. The results are compared with the velocity field. The error is represented as $\sqrt{\frac{MSE}{\sum_i\sum_j (p_x^2 + p_y^2)}}$, where $MSE = \sum_i\sum_j[(p_x(i,j) - v_x(i,j))^2 + (p_y(i,j) - v_y(i,j))^2]$, (i,j) are the image coordinates, (p_x, p_y) is the velocity field, (u,v) is the optical flow of the experiment. Since the Lucas and Kanade's method has been proven to be the most reliable method of computing the optical flow [8]. We would only compare the approach using various moment with it. In the following table Aij represents the method using the moment with i,j as its order and repetition.

Table1: Performance of optical flow algorithm for different type of motion

		E	A00	A11	A20	A02	A40	A04
Transl- -ation	$\Delta X = 1$	0.001268	0.001268	0.001268	0.001268	0.001268	0.001268	0.001268
	$\Delta Y = 1$	0.001268	0.001268	0.001268	0.001268	0.001268	0.001268	0.001268
	$\Delta X = \Delta Y = 1$	0.049415	0.044223	0.026338	0.038241	0.037177	0.056696	0.081236
Rotati- -on	$\Omega_3 = 0.05$	1.904222	1.853218	1.774374	1.752585	1.764074	1.731297	1.724257
	$\Omega_3 = 0.1$	2.314736	2.246103	2.040198	2.012155	2.024910	1.991816	1.993311
	$\Omega_3 = 0.15$	4.707632	2.993272	3.298785	3.070348	3.231489	2.833572	2.625811
Expan- -sion	$\Delta Z = 15$	1.888912	1.881496	1.819882	1.836890	1.815774	1.856602	1.902494
	$\Delta Z = 20$	1.925519	1.905521	1.843921	1.805559	1.833438	1.800499	1.840901
	$\Delta Z = 25$	1.987802	1.932241	1.896406	1.831599	1.875132	1.801571	1.822336

The table above shows how the integrated invariant feature method is better than the traditional one. The high order invariant feature is much good for rotation and expansion and the low order is better for translation, but it requires more computation. Therefore in the real application, we should trade off between the accuracy and computation cost.

VI. CONCLUSION

In this paper, we present a general idea of improving the accuracy of the optical flow constraint.

We use the feature M and A based on local transformation instead of the brightness. Three properties are defined to guarantee the goodness of the model based on invariant feature. A special choice of such feature is discussed. The feature is proved to be more stable than brightness and has close relationship to the Nagel's image brightness change model. The Zernike moment is applied to resist the noise effect. The experiment results show that the model based on the invariant feature is better than brightness based optical flow constraint, and better accommodates the change of brightness.

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