Multiple Fault Diagnosis of Analog Circuits Based on Large Change Sensitivity Analysis

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Abstract **- A new method is proposed in this paper for multiple fault diagnosis in linear analog circuits. The test equation in the traditional large change sensitivity method is modified to establish the linear relationship between the measurements and the faulty parameter deviations with the coefficient matrix derived from the nominal values of circuit parameters. By using a recently developed method to seek the ambiguity groups in the test equation, the faulty parameters with minimum number of solutions satisfying the test equation can be located. Exact parameter deviations are obtained from the test equation. An example circuit is provided to illustrate the proposed method.**

1 Introduction

Analog fault diagnosis usually consists of three stages which respectively address three important problems in the analog testing and diagnosis: fault detection to find out if the circuit under test (CUT) is faulty, fault location to identify where the faulty parameters are, and parameter evaluation to tell how much of the parameter deviations. Since the middle of the 1970s, analog fault diagnosis attracted much attention from test engineers and academic researchers. A detailed summary of the research efforts for the pre-1990 period is given respectively in [1] and [2]. With today's rapid development of analog VLSI circuits and mixed-signal systems, analog fault diagnosis gains more interests [3-5]. This research interests are fully justified since analog testing lags far behind the mature digital testing techniques due to its inherited bottlenecks such as nonlinearity, tolerance, lack of efficient fault models and limited accessibility.

Large change sensitivity analysis is one of test verification methods used for analog fault diagnosis [6]. Its significant advantage is that both the parametric and catastrophic faults can be exactly obtained. In the Simulation-Before-Test methods [1], large change sensitivity methods are typically utilized to construct the fault dictionary. This leads to the large computational requirements. In this paper, a new approach is proposed to combine the large change sensitivity method with the ambiguity group locating technique, thus expand the application of large change sensitivity method into the Simulation-After-Test category, which greatly reduces the computation cost.

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2 Establishment of Test Equation

Assume that the nominal parameter values and the topology of the CUT with *n+1* nodes and *p* parameters are known, modified nodal equation [7] to describe the circuit with nominal parameters is

$$T_0 X_0 = W_0 \tag{1}$$

where T_0 is a *gxg* coefficient matrix, X_0 is a *gx1* vector of node voltages and/or parameter currents, and W_0 is a *gx1* excitation vector. Note that $g \geq n$ in modified nodal analysis.

The solution vector X_0 is obtained by inverting the non-singular matrix T_0:

$$X_0 = T_0^{-1} W_0 \tag{2}$$

Suppose that *f* of *p* parameters are faulty from their nominal values $h_{10}, h_{20}, \ldots, h_{f0}$ to the new values $h_{10} + d_1, h_{20} + d_2, \ldots, h_{f0} + d_f$, where $d_1, d_2, \ldots, d_f$ are the amounts of parameter changes. The new values of circuit parameters are:

$$h_n = h_{n0} + d_n \qquad n = 1, 2, \ldots, f \tag{3}$$

Assume that all faulty parameters have their corresponding changes in the coefficient matrix in the form $p_n d_v q_n^t$ with

$$\begin{aligned} p_n &= e_{n_i} - e_{n_j} \\ q_n &= e_{n_k} - e_{n_l} \qquad n = 1, 2, \ldots, f \end{aligned} \tag{4}$$

where superscript *t* represents the transpose of vector/matrix and e_v represents a *gx1* vector of zeros except for the v^{th} entry, which is equal to one.

The equation describing the circuit with the faulty parameters under the same excitation vectors are

$$TX = \left(T_0 + \sum_{n=1}^{f} p_v d_n q_n^t \right) X = W_0 \tag{5a}$$

or

$$\left(T_0 + P_f \, diag\,(d)\, Q_f^{\,t} \right) X = W_0 \tag{5b}$$

Here P_f and Q_f are *gxf* matrices which contain 0 and ± 1 entries:

$$\begin{aligned} P_f &= \lfloor p_1 \;\; p_2 \;\; \ldots \;\; p_f \rfloor \\ Q_f &= \lfloor q_1 \;\; q_2 \;\; \ldots \;\; q_f \rfloor \end{aligned} \tag{6}$$

and $diag(d)$ is an *fxf* diagonal matrix while d is an *fx1* vector:

$$d = [d_1 \; d_2 \; \ldots \; d_f]^t \tag{7}$$

Equation (5b) can be written as

$$\left(T_0 + P_f \ diag(d) Q_f^{\ t}\right)\left(X_0 + \Delta X\right) = W_0 \qquad (8)$$

where

$$X = X_0 + \Delta X \qquad (9)$$

After substituting (1) into (8), following equation is established:

$$\Delta X = -T_0^{-1} P_f \ diag(d) Q_f^{\ t} X \qquad (10)$$

Assume that the faulty parameter $F_n \ (v = 1, 2, \dots, f)$ is located on intersection of rows i_v and j_v and columns k_v and l_v of coefficient matrix T, then matrices P_f and Q_f in (6) have the following forms considering (4):

$$\begin{aligned} P_f &= [e_{i_1} - e_{j_1} \ \ e_{i_2} - e_{j_2} \ \dots \ e_{i_f} - e_{j_f}] \\ Q_f &= [e_{k_1} - e_{l_1} \ \ e_{k_2} - e_{l_2} \ \dots \ e_{k_f} - e_{l_f}] \end{aligned} \qquad (11)$$

Let us denote a *gxg* matrix S_0 as follows

$$S_0 = [s_1 \ s_2 \ \dots \ s_g] = -T_0^{-1} \qquad (12a)$$

and re-write vector X in following form:

$$X = [x_1 \ x_2 \ \dots \ x_g] \qquad (12b)$$

where $s_v \ (v = 1, 2, \dots, g)$ is an *gx1* vector while $x_v \ (v = 1, 2, \dots, g)$ is a scalar.

Thus the products of S_0 and P_f, Q_f^t and X can be written as

$$\begin{aligned} S_{GF} &= S_0 P_f = S_0 [e_{i_1} - e_{j_1} \ \ e_{i_2} - e_{j_2} \ \dots \ e_{i_f} - e_{j_f}] \\ &= [s_{i_1} - s_{j_1} \ \ s_{i_2} - s_{j_2} \ \dots \ s_{i_f} - s_{j_f}] \\ Q_f^{\ t} X &= [e_{k_1} - e_{l_1} \ \ e_{k_2} - e_{l_2} \ \dots \ e_{k_f} - e_{l_f}]^t X \\ &= [x_{k_1} - x_{l_1} \ \ x_{k_2} - x_{l_2} \ \dots \ x_{k_f} - x_{l_f}]^t \end{aligned} \qquad (13)$$

where G indicates the set of all variables in solution vector X, and the **fault set** F represents the set of all the faulty parameters.

Denote an *fx1* vector

$$l_F = diag(d) \ Q_f^{\ t} X \qquad (14)$$

and consider (7) and (13)

$$\begin{aligned} l_F &= diag(d) \ Q_f^{\ t} X \\ &= diag(d) [x_{k_1} - x_{l_1} \ \ x_{k_2} - x_{l_2} \ \dots \ x_{k_f} - x_{l_f}]^t \\ &= [d_1 (x_{k_1} - x_{l_1}) \ \ d_2 (x_{k_2} - x_{l_2}) \ \dots \ d_f (x_{k_f} - x_{l_f})]^t \end{aligned} \qquad (15)$$

Thus (10) can be re-written as

$$\Delta X = S_{GF} \ l_F \qquad (16)$$

Assume that the first m elements of ΔX can be measured and $f \le m - 1 \le p$, we obtain

$$\begin{bmatrix} \Delta X^{\ M} \\ \Delta X^{\ G-M} \end{bmatrix} = \begin{bmatrix} S_{MF} \\ S_{G-M,F} \end{bmatrix} l_F \qquad (17)$$

where M represents the set of measurements. Hence, following equation is obtained:

$$\Delta X^M = S_{MF} \ l_F \qquad (18)$$

Here S_{MF} is an *mxf* matrix whose columns correspond to the faulty parameters in the circuit. Similarly S_{MP} is an *mxp* matrix whose columns corresponding to all of the parameters in the circuit, where P indicates the set of all parameters. Equation (18) is called the **test equation** and matrix S_{MP} is called the **test matrix**.

3 Fault Diagnosis

If the vector ΔX^M is a zero vector, it means that the CUT is fault-free, i.e., no faults can be detected by the existing measurements. Otherwise, the CUT is judged to be faulty.

For the purpose of fault location, we need to find out the sets of columns in the test matrix S_{MP} that satisfy (18) with fault set F having the minimum size. Recent proposed ambiguity group locating techniques [8-9] can be utilized to implement this objective, providing that the method is modified as steps 2-7 of algorithm in Section 4:

Both of the proposed methods in [9] and this paper utilize similar ambiguity groups locating technique with different test equations. The method proposed in [9] is based on the nodal analysis and the faulty current nodes are first located by ambiguity group locating technique, and then the faulty parameters are located by using incident signal matrix. The method proposed in this paper is based on the large change sensitivity analysis, and the faulty parameters are located directly.

After location of the fault set F with the minimum size, vector l_F is obtained by solving (18) assuming that S_{MF} is a full column rank matrix:

$$l_F = \left(S_{MF}^{\ t} S_{MF}\right)^{-1} S_{MF}^{\ t} \Delta X^M \qquad (19)$$

The full vector ΔX can be computed by (16) since matrix S_{GF} and vector l_F are known. The solution vector X is consequently determined by (9). Finally the parameter deviations d can be obtained by (15):

$$d = \left[\frac{l_1}{x_{k_1} - x_{l_1}} \ \ \frac{l_2}{x_{k_2} - x_{l_2}} \ \ \dots \ \ \frac{l_f}{x_{k_f} - x_{k_f}} \right]^t \qquad (20)$$

4 The Algorithm for Fault Diagnosis

The following procedure is recommended to implement the proposed method:

Step 1: Measure the CUT and the fault-free sample circuit under the same excitations through the same selected nodes and/or parameters. If the resulting vector ΔX^M is a zero vector, the CUT is concluded as fault-free. Otherwise, at least one faulty parameter exists in the CUT.

Step 2: Append the vector ΔX^M to matrix S_{MP} to construct a new *mx (p+1)* matrix with ΔX^M being its first column, then normalize and eliminate the first column ΔX^M from the new matrix by Gauss elimination step and obtain *(m-1)xp* test verification

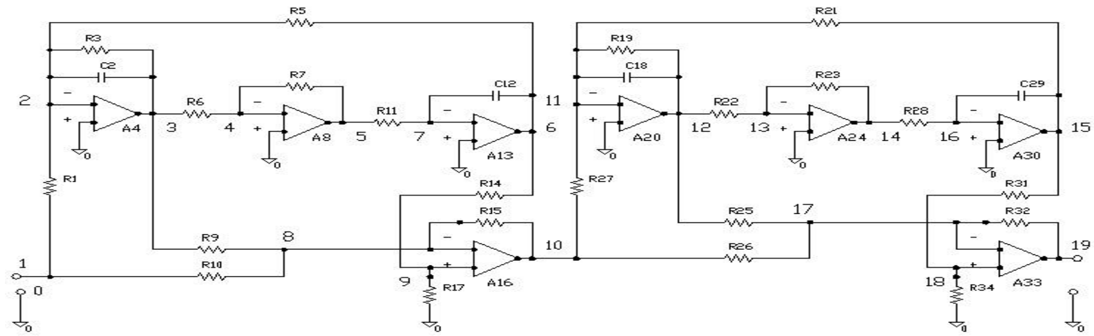

Figure 1(a) Active lowpass filter;

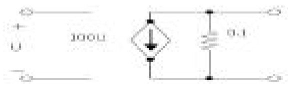

Figure 1 (b) Model of OPAMP.

matrix. Apply the QR factorization step to the test verification matrix to form a linear combination matrix *C*.

Step 3: Locate all of the suspicious fault sets *F* by analyzing all columns with zero-element in matrix *C*, then apply the Lemma 2 [9] to exclude the contradicted fault sets and find out the minimum size of *F* among the remaining fault sets, ***min(size(F))***;

Step 4: Usually several suspicious fault sets have the same size, ***min(size(F))***. Select one of them arbitrarily for analysis. Only one element in the selected *F* is from the co-basis and the remaining elements from the basis. Swap that co-basis element which corresponds to column *k* in matrix *C* with one of basis elements which corresponds to row *j* in the matrix *C*. There are two rules for swapping, one is that row *j* is selected to make sure the resulting entry c_{jk} on the intersection of row *j* and column *k* of matrix *C* should be non-zero; Another rule is that if one element in the current basis has been swapped into the basis by the previous swapping performance, then this element will not be considered during the later swapping performance.

Step 5: New matrix *C* is obtained after swapping, then locate new fault sets *F* satisfying Lemma 2 [9] and find out new value of ***min(size(F))***.

Step 6: (a) If the new value of ***min(size(F))*** is less than the old value before swapping, record all *F* with size ***min(size(F))*** and go to step 4;

(b) If not, and there is still basis element which has not been swapped once, keep the selected co-basis element which correspond to column *k* in matrix *C*, but change the selected basis element corresponding to row *j* to the next element also contained in *F*. Go to step 5.

(c) If all of the elements in the original basis have already been swapped once, and the value of ***min(size(F))*** can not be decreased any more by swapping, go to step 7.

Step 7: Take the *F* with minimum size ***min(size(F))*** and check the difference between the measurements and the signal derivation values computed by (19) and (16). If this difference is within a preset tolerance limit then *F* is used to determine faults, otherwise next larger *F* is used; If none *F* satisfies the tolerance requirement, take the adjoint *F* in the list with larger size and check the tolerances. If none *F* in the list satisfy tolerance requirements, conclude that no solution to the test equation and stop.

Step 8: Compute vector l_F by (19), then ΔX by (16), solution vector X by (9) and finally the parameter deviations d by (20).

5 Analog Example Circuit

An active lowpass filter [10] shown in Fig. 1a is provided to illustrate the approach proposed in the paper. The example circuit has 20 nodes and 22 resistors, 4 capacitors, and 8 amplifiers with the following nominal values (all resistors in $k\Omega$ and capacitors in mF):

R_1=0.182, C_2=0.01, R_3=1.57, R_5=2.64, R_6=10.0, R_7=10.0, R_9=100.0, R_{10}=11.1, R_{11}=2.64, C_{12}=0.01, R_{14}=5.41, R_{15}=1.0, R_{17}=1.0, C_{18}=0.01, R_{19}=4.84, R_{21}=2.32, R_{22}=10.0, R_{23}=10.0, R_{25}=500.0, R_{26}=111.1, R_{27}=1.14, R_{28}=2.32, C_{29}=0.01, R_{31}=72.4, R_{32}=10.0, R_{34}=10.0. The model of operational amplifiers is in Fig.1(b). The current source is $j(t) = 10^{-2}\cos(2000t)A$.

Assume that the faulty parameters are R_6 which was changed from 10.0 $k\Omega$ to 20.0 $k\Omega$ and R_{26} changed from 111.1 $k\Omega$ to 75.0 $k\Omega$. The corresponding admittance deviations are $\Delta G_6 = -5.0e-5\ S$ and $\Delta G_{26} = 4.3324e-6S$. The nodal voltage measurements are on nodes {2, 5, 7, 8, 11, 17, 19}. Hence *n=19*, *p=42*, *f=2*, *m=7* and $f \le m-1 \le p$. The vector of measured changes of nodal voltage ΔX^M is non-zero, thus indicating the fault(s) detected in the CUT. In step 2 of section 4, a *6x28* matrix *C* is obtained by QR factorization with its basis {31, <u>26</u>, 11, 17, 23, 10} and co-basis {7, 8, 9, <u>6</u>, 3, 12, 13, 14, 15, 16, 4, 18, 19, 20, 21, 22, 5, 24, 25, 2, 27, 28, 29, 30, 1, 32, 33, 34}.

According to step 3, 18 suspicious faulty groups are obtained and 16 of them are qualified with Lemma 2. Only one suspicious faulty group F={6, 26} was found with ***min(size(F))***=2.

Based on the swapping principles in step 4, and following the steps 5-7, we can not reduce the value of ***min(size(F))***. Thus F={6, 26} is our only solution located by the procedures in section 4, which is the exact solution for the given CUT.

Equation (18) thus has the following form:

$$\begin{bmatrix} -0.0011-0.0095i \\ 0.1141+0.0053i \\ 0.0013+0.0151i \\ 0.0296+0.3385i \\ 0.0003+0.0000i \\ -0.0089-0.0969i \\ 0.2294-0.1847i \end{bmatrix} = 10^4 \times \begin{bmatrix} -0.0071+0.0006i & -0.0000-0.0000i \\ 0.0070-0.0854i & -0.0000+0.0000i \\ 0.0114-0.0006i & -0.0000+0.0000i \\ 0.2544-0.0132i & 0.0001+0.0000i \\ 0.0000-0.0002i & 0.0000-0.0000i \\ -0.0727+0.0053i & -0.0068+0.0000i \\ -0.1068+0.0079i & -1.0076+0.0000i \end{bmatrix} l_F$$

and by (19),

$$l_F = 10^{-3} \times \begin{bmatrix} 0.0047 + 0.1333i \\ -0.0243 + 0.0042i \end{bmatrix}$$

The full vector ΔX can be computed by (16), and the solution vector X is consequently determined by (9). Finally the corresponding parameter deviation values computed by (20) are

$$\begin{bmatrix} \Delta G9 \\ \Delta G37 \end{bmatrix} = \begin{bmatrix} -0.5000 \text{ e} - 4 \\ 0.0433 \text{ e} - 4 \end{bmatrix}$$

which are the exact deviation values of the faulty elements R_6 and R_{26}.

6 Conclusions

Combinatorial searches of suspicious faulty parameters [1] for multiple-fault diagnosis are computationally expensive. In this paper a new multiple-fault diagnosis method is proposed based on the large change sensitivity analysis and identification of the ambiguity groups in the test equation for the linear analog circuit. The test equation establishes linear relationship between the measured circuit responses and the faulty parameter deviations. The number of voltage and/or current measurements minus one is no more than the number of parameters while no less than the number of faulty parameters. Any difference of the measurements on the same nodes and/or parameters between the CUT and fault-free sample circuit with the same excitations will indicate that the CUT is faulty. A recently developed approach based on the QR factorization technique is utilized to identify ambiguity groups in the test equation. This yields a numerically efficient search for the sets of candidate faulty parameters. Faulty parameters can be identified in the number of operations $O(p^3)$ rather than $O\left(\sum_{i=1}^{f}\binom{p}{i}\right)$ required for combinatorial searches, which is a significant improvement in computational efficiency. Finally faulty parameters are evaluated based on the analysis results during the establishment of the test equation. The proposed methods can provide the exact solution to all of the circuit parameters. The parameters can change from zero to infinity. The measurements can be voltages or currents and the circuit is excited once.

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