MULTIPLE FAULT DIAGNOSIS OF ANALOG CIRCUITS BY LOCATING AMBIGUITY GROUPS OF TEST EQUATION

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ABSTRACT

This paper proposes a method to diagnose the multiple faults in the linear analog circuits. Test equation establishes the relationship between the measured responses and faulty excitations due to faulty elements. The QR factorization is applied to identify ambiguity groups in the test verification matrix. The suspicious faulty excitations of the minimum size are determined. Faulty parameters are evaluated using the structural incident signal matrix. Finally this method is illustrated with an example circuit.

1. INTRODUCTION

Fault diagnosis is an important problem of analog circuit testing. Due to the lack of efficient fault models and limited accessible nodes, and due to the inherent features of analog circuits such as component tolerance and nonlinearity, the automation level of analog fault diagnosis has not yet achieved the same level of its digital counterpart. During the past years, many efforts have been devoted to this area [1-4] and interests in these subjects are continuing now [5-7]. Fault diagnosis methods are generally divided into simulation-before-test (SBT) methods and simulation-after-test (SAT) methods [4]. SBT methods are usually based on the fault dictionary techniques that require costly simulation and extensive data buses. Parameter identification methods under the SAT category can identify all the parameters of the network if sufficient measurements are guaranteed. They require sufficient number of accessible nodes and may only provide approximate parameter solutions due to linearization. When the number of measurements is less than the number of elements or nodes of the network, but is greater than the number of faults, fault verification methods under the SAT category are used. The fault verification methods can provide the exact solution to the network parameters and can be applied to detect large parameter changes. The network is excited once and only voltage measurements are performed.

Most of efforts in this area are paid to the single-fault diagnosis of the analog circuits. In [8], multiple port approach was implemented by checking the consistency or inconsistency of suitable sets of linear equations, with current excitations applied to all measurement ports successively. In [9], multiple-fault location is based on the decomposition technique and the same current excitation is required for different accessible ports. The method presented in this paper extends the verification methods based on the nodal analysis [10] for linear test equations and provides procedures to detect the faulty circuits, locate the faulty elements and identify the faulty parameters. The method enhances fault verification techniques with efficient search for faulty nodes based on the QR factorization.

2. EQUATIONS FOR FAULTY NETWORK

Let us assume that the network under test has *n+1* nodes and *m* test points (current or voltage measurements) and $f<m$ is the number of faults in the network. The modified nodal equations for the nominal values of the elements have the form

$$TX = W \tag{1}$$

where T is an $n \times n$ coefficient matrix, X is the vector of nodal voltages and parameter currents, and W is the excitation vector.

For the faulty network, assuming the same excitations, we obtain

$$(T+\Delta T)(X+\Delta X) = W \tag{2}$$

Thus

$$T\,\Delta X = -\Delta T\,\hat{X} \tag{3}$$

$$\hat{X} = X + \Delta X \tag{4}$$

where $\hat{X}$ is the solution vector for the faulty network. We can compute ΔX assuming that T is nonsingular and obtain

$$\Delta X = -T^{-1}\Delta T\,\hat{X} \tag{5}$$

Let us denote

$$\Delta W = -\Delta T\,\hat{X} \tag{6}$$

ΔW represents changes in excitations caused by faulty elements and we call it the faulty excitations. The corresponding nodes or parameters are faulty. Similarly, nodes or parameter with zero faulty excitations are fault-free. The equation (5) becomes

$$\Delta X = T^{-1}\Delta W \tag{7}$$

We can assume that a few elements are faulty, in which case ΔW has the form

$$\Delta W = \begin{bmatrix} 0 \\ \Delta W^{F} \\ 0 \end{bmatrix} \tag{8}$$

Assuming that the first *m* elements of *X* can be measured we obtain

$$\begin{bmatrix} \Delta X^{M} \\ \Delta X^{N-M} \end{bmatrix} = T^{-1}\begin{bmatrix} 0 \\ \Delta W^{F} \\ 0 \end{bmatrix} \tag{9}$$

N indicates the set of all equations, *M* the set of measurements (test nodes). Hence,

$$\Delta X^{M} = B_{MF}\,\Delta W^{F} \tag{10}$$

where

$$T^{-1} = \begin{bmatrix} B_{M1} & B_{MF} & B_{M2} \\ B_{N-M,1} & B_{N-M,F} & B_{N-M,2} \end{bmatrix} \tag{11}$$

Equation (10) has to be satisfied when the set *F* includes all circuit excitations associated with faulty elements in the network.

The columns in B_{MF} correspond to faulty nodes or faulty parameters in the circuit. Our aim is to find out the sets of columns in *B* that satisfy equation (10) with the minimum number of faults, that is, vector ΔW^{F} has the minimum numbers of nonzero values.

Recent results [11] to identify the ambiguity group of the test matrix are utilized to implement our aim.

Since the measurement set M is known and set of faulty excitations F is not known, we will solve our fault diagnosis problem as the following ambiguity group problem: find minimum form ambiguity group F in the set of columns of the **test matrix** B_M which satisfies the **test equation** (10).

3. AMBIGUITY GROUPS IN THE TEST EQUATIONS

Let us assume that the test equations were formulated and that the faulty excitations ΔW are related to test measurements ΔX^M through the test matrix B_M as follows:

$$\Delta X^M = B_M \, \Delta W \tag{12}$$

Where B_M is an $m \times n$ matrix, ΔW is an $n \times 1$ vector and ΔX^M is an $m \times 1$ vector.

Denote an augmented $m \times (n+1)$ matrix B_S as the concatenation of the vector ΔX^M and the matrix B_M:

$$B_S = [\Delta X^M \;\; B_M] \tag{13a}$$

We will normalize the first column of matrix B_S to have a unit in its first row, then eliminate the remaining elements of the first row of matrix B_S performed in a similar way to Gaussian elimination step as follows:

$$\hat{B}_S(i, j) = B_S(i, j) - \frac{B_S(i,1)}{B_S(1,1)} B_S(1, j), \quad i = 1, 2, ..., n; \; j = 2, 3, ..., m+1 \tag{14}$$

The obtained matrix $\hat{B}_S$ has the following form:

$$\hat{B}_S = \begin{bmatrix} (\Delta X_1)^{1,1} & 0^{1,n} \\ (\Delta X_2)^{m-1,1} & B^{m-1,n} \end{bmatrix}, \quad m-1<n \tag{13b}$$

Note that the superscript represents the size of the vector or matrix. Matrix B is obtained from B_M after elimination of dependence on ΔX^M and is called **test verification matrix**. Next we will analyze the dependences among the columns of the test verification matrix B. The dependences of the desired columns of matrix B surely indicate the dependences between ΔX^M and the corresponding columns of matrix B_M.

The rank of B determines a maximum number of faults that can be uniquely identified by solving the test equations. Because $m-1<n$, B can be written as

$$B = B_1 [I \;\; C] \tag{15}$$

Where $(m-1) \times r$ matrix B_1 has the full column rank equal to the rank r of the matrix B, and $(m-1) \times (n-r)$ matrix C expand the dependent columns of B into a set of the basis columns B_1. Note that the selection of independent columns B_1 is not unique and is an important issue in solving the test equations in the presence of ambiguities. Different partitions define different linear combination matrices C.

In order to efficiently find such a partition for any ambiguity group or its combination, we will look for a partition (15) with the matrix C in a minimum form, where matrix C, is in a minimum form if one or several of its columns has the maximum number of coefficients equal to zero.

Here we will refer to a numerically robust solution algorithm based on the *QR* factorization [11]. The *QR* algorithm finds a numerically stable solution of over determined system of linear equations that minimizes the least square error.

As a result of the *QR* factorization of $(m-1) \times n$ test verification matrix B, we can formulate the following equation:

$$BE = QR \tag{16}$$

Where E is $n \times n$ column selection matrix, Q is $(m-1) \times (m-1)$ orthogonal matrix, and R is $(m-1) \times n$ upper triangular matrix. Matrix E has only a single nonzero element equal to one in each column. Matrix product BE represents a permutation of the original columns of the test verification matrix B. Matrix R has its rank equal to the rank of test matrix B. Since R is an upper triangular matrix and $m-1<n$, R can be written as

$$R = \begin{bmatrix} R_1 & R_2 \\ 0 & 0 \end{bmatrix} \tag{17}$$

Where R_1 is $r \times r$ upper triangular and has its rank equal to the rank of the test verification matrix B.

The following theorem [11] provides a basis for a numerically efficient approach to finding the ambiguity groups.

Theorem:

A linear combination matrix C can be numerically obtained from the QR factorization of the test verification matrix B using

$$C = R_1^{-1} R_2 \tag{18}$$

To find the minimum form partition, we have to swap an element of the basis with an element of the co-basis in the ambiguity cluster in order to increase number of nonzero coefficients in C.

Lemma 1 [11]:

The necessary condition for swapping to increase the number of zero coefficients in C is that the columns of basis and co-basis to be swapped have a singular 2x2 submatrix of nonzero coefficients.

Let us consider a linear combination matrix C with a singular submatrix $[c_{jk}, c_{jm}; c_{ik}, c_{im}]$ with all nonzero coefficients. The j^{th} column of the corresponding co-basis $B_{2j} \in B_2$ is related to columns of the basis B_1 through the j^{th} column of C as follows:

$$B_{2j} = \sum_{i=1}^{r} c_{ij} \, B_{1i} \tag{19}$$

where $B_{1i} \in B_1$ is the i^{th} column of the basis. Let us consider a nonzero coefficient c_{jk} of the k^{th} column of C in a minimum form. If we swap the j^{th} element of the basis with k^{th} element of the co-basis, then

$$B_{1j} = \frac{1}{c_{jk}} B_{2k} - \sum_{i \neq j} \frac{c_{ik}}{c_{jk}} B_{1i} \tag{20}$$

In addition, all other columns of the co-basis will be equal to

$$B_{2m} = \sum_{i \neq j} \left(c_{im} - \frac{c_{jm} c_{ik}}{c_{jk}} \right) B_{1i} + \frac{c_{jm}}{c_{jk}} B_{2k} \tag{21}$$

After swapping, all zero locations in the k^{th} column of C will remain zero as they were in the original C. However, as can be deducted from (21), a nonzero location c_{im} in column m and row i will become zero.

Any column of C with zero coefficients form an ambiguity group F and has to be consider for further processing.

Lemma 2:

A necessary condition for an ambiguity group F of the linear combination matrix C to contain the set of all faults in the tested circuit is that the rank of the corresponding columns in the original test matrix is equal to the cardinality of F.

$$rank(B_{MF}) = card(F) \qquad (22)$$

Thus according to Lemma 2 any ambiguity group of test verification matrix which do satisfy (22) does not have to be further analyzed.

4. COMPUTATION OF FAULTY ELEMENTS

After location of faulty excitations, the deviation of the excitation vector can be derived by solving test equation (10). Assuming that B_{MF} is a full column rank matrix:

$$\Delta W^F = \left(B_{MF}{}^T B_{MF}\right)^{-1} B_{MF}{}^T \Delta X^M \qquad (23)$$

Where superscipt T denotes transpose of matrices and vectors.

Then the deviation of the excitation vector can be obtained by filling out the remaining elements with zeros:

$$\Delta W = \begin{bmatrix} 0 \\ \Delta W^F \\ 0 \end{bmatrix} \qquad (24)$$

The deviation of the solution vector and entire solution vector are computed from:

$$\Delta X = T^{-1} \Delta W \qquad (7)$$

$$\hat{X} = X + \Delta X \qquad (4)$$

In order to compute the deviation of faulty elements ΔT_b, the incident matrices P and Q of the circuit are utilized:

$$\Delta T = P\, diag(\Delta T_b) Q^T \qquad (25)$$

Note that ΔT is an $n \times n$ matrix, but ΔT_b is an $p \times 1$ vector.

Combining this equation into

$$\Delta W = -\Delta T\, \hat{X} \qquad (26)$$

We can get

$$\Delta W = -P\, diag(\Delta T_b) Q^T\, \hat{X} = X_{inc} \Delta T_b \qquad (27)$$

where $X_{inc} = -P\, diag(Q^T \hat{X})$ is called the incident signal (voltages and currents) matrix.

Assuming that k of p elements are faulty and f of n of excitations have faulty excitations, we re-arrange the equation (27) as follows:

$$X_{inc}{}^{f,k}\ (\Delta T_b)^k + X_{inc}{}^{f,p-k}\ 0^{p-k} = (\Delta W)^f \qquad (28a)$$

$$X_{inc}{}^{n-f,k}\ (\Delta T_b)^k + X_{inc}{}^{n-f,p-k}\ 0^{p-k} = 0^{n-f} \qquad (28b)$$

Here the superscript indicates the size of the matrix or vector. The equation (28b) is worth consideration. Obviously with nonzero values of $\Delta T_b{}^k$, $X_{inc}{}^{n-f,k}$ must be $0^{n-f,k}$ with probability equal to 1. We can obtain the position of faulty elements $(\Delta T_b)^k$ from the solution of equation (28b) as follows:

Lemma 3:

The k faulty elements are included in the element sets whose corresponding columns have all zero elements in the matrix $X_{inc}{}^{n-f,p}$.

The faulty elements then can be derived by solving (28a):

$$(\Delta T_b)^k = \left(\left(X_{inc}{}^{f,k}\right)^T X_{inc}{}^{f,k}\right)^{-1} (X_{inc}{}^{f,k})^T\ (\Delta W)^f \qquad (29)$$

assuming that $f \geq k$ and that $X_{inc}{}^{f,k}$ has full column rank.

5. EXAMPLE CIRCUIT

To illustrate the approach proposed above, let us discuss an example of a linear circuit described by nodal equations with the number of measurements less than the number of nodes, but greater than the number of faulty nodes.

Example:

The resistive network shown in Figure 1 has 21 nodes and 39 resistors with the following nominal values (all resistors in Ω): R1=2.125, R2=3.6, R3=4.7, R4=11.5, R5=12.6, R6=21.2, R7=3.7, R8=0.54, R9=3.54, R10=3.125, R11=6.6, R12=5.7, R13=19.5, R14=12.8, R15=12.2, R16=3.2, R17=1.54, R18=8.7, R19=2.27, R20=3.16, R21=41.7, R22=31.5, R23=22.6, R24=51.2, R25=13.7, R26=3.44, R27=13.4, R28=31.9, R29=16.1, R30=11.7, R31=11.5, R32=17.8, R33=22.2, R34=23.2, R35=11.4, R36=18.7, R37=3.12, R38=33.2, R39=8.67. The current source is $J = 1A$.

Assume that the faulty elements are R9, which was changed from 3.54Ω to 7.9Ω, and R37 changed from 3.12Ω to 2.8Ω. The admittance deviations of faulty elements are $\Delta G9 = 1/7.9 - 1/3.54 = -0.1559/\Omega$ and $\Delta G37 = 1/2.8 - 1/3.12 = 0.03663/\Omega$. Obviously there are 3 faulty nodes here: nodes (1, 14, 16). The measurement nodes are nodes (2, 4, 15, 16, 17). The measured changes of nodal voltage are:

$$\Delta X^M = \begin{bmatrix} 1.1593e+000 \\ 1.1905e+000 \\ 4.8116e-002 \\ 4.3046e-002 \\ 5.0607e-002 \end{bmatrix}$$

After QR factorization, the order of columns is (15, 16, 4, 17, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 1, 2, 3, 18, 19, 20). So the basis is (*15*, 16, 4, 17) and co-basis is (5, 6, 7, 8, 9, 10, 11, 12, 13, *14*, *1*, 2, 3, 18, 19, 20).

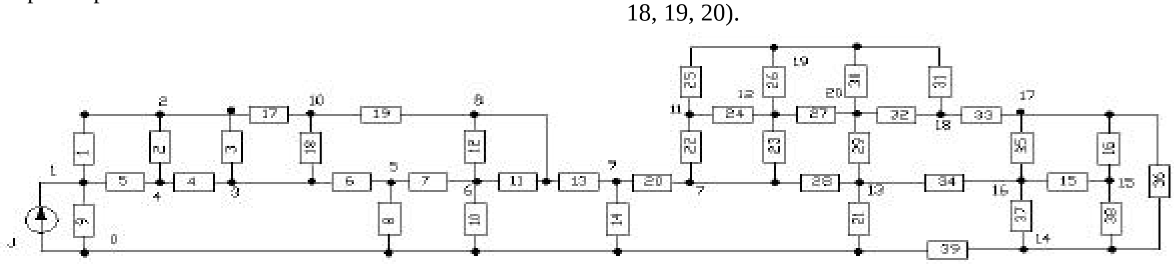

Figure 1. Resistive network example

Analyzing the obtained matrix C, we find out the 10th column (node 14) and the 11th column (node 1) in matrix C having one zero element,

$$C(\ldots 10^{th}\ column\quad 11^{th}\ column\ \ldots) = \begin{bmatrix} 5.7985e-002 & -7.3919e-005 \\ 6.1702e-001 & 4.8821e-004 \\ 2.1778e-016 & 7.2769e-016 \\ 1.0295e-001 & -1.3124e-004 \end{bmatrix}$$

The non-zero elements in the 10th column are in rows (1 2 4), so the corresponding nodes in basis are (15 16 17), and the corresponding ambiguity group is *F=(14 15 16 17)*. The corresponding ambiguity group for the 11th column of C is *F=(1 15 16 17)*.

Checking the rank of the corresponding columns in the original test matrix B_M, we obtain the following results: $rank(B_{MF}) = 3$ for the first suspicious solution set while $rank(B_{MF}) = 4$ for the second suspicious solution. By Lemma 2, the first set does not contain all the faults in the circuit and the second set *F=(1 15 16 17)* may provide a solution to the fault verification problem.

Swapping the first column of basis (node 15) with the 10th column of co-basis (node 14), we got the new matrix C with two zeros in the 11th column corresponding to node 1.

$$C(\ldots 11^{th}\ column\ \ldots) = \begin{bmatrix} -1.2748e-003 \\ 1.2748e-003 \\ 7.2797e-016 \\ -5.3316e-017 \end{bmatrix}$$

The corresponding ambiguity group is the node set *F=(14, 16, 1)*, which are the exact faulty nodes of the circuit. This set satisfies Lemma 2 with $rank(B_{MF}) = 3$ and yields a minimum number of faulty parameters consistent with test equation (10).

Thus equation (10) has the following form:

$$\begin{bmatrix} 1.1593e+000 \\ 1.1905e+000 \\ 4.8116e-002 \\ 4.3046e-002 \\ 5.0607e-002 \end{bmatrix} = \begin{bmatrix} 1.8790e+000 & 8.8388e-002 & 1.1056e-001 \\ 1.9296e+000 & 8.2372e-002 & 1.0303e-001 \\ 7.9465e-002 & 6.0763e+000 & 7.2382e+000 \\ 7.2322e-002 & 6.2274e+000 & 8.2319e+000 \\ 8.3425e-002 & 5.9732e+000 & 7.0749e+000 \end{bmatrix} \Delta W^F$$

According to Equation (23), the deviations of faulty excitations are:

$$\Delta W^F = \begin{bmatrix} 6.1700e-001 \\ 7.8655e-004 \\ -7.8655e-004 \end{bmatrix}$$

To locate the faulty elements, we compute the incident signal matrix and locate its zero sub-matrix $X_{inc}^{n-f,p}$. Analyzing the matrix X_{inc} as required by Lemma 3, we found out that the columns of the incidence signal matrix which correspond to the elements (R9, R37, R39) has zeros in *n-f* rows, which means that (R9, R37, R39) are identified as suspicious elements. The corresponding parameter deviation values computed from equation (29) are

$$\begin{bmatrix} \Delta G9 \\ \Delta G37 \\ \Delta G39 \end{bmatrix} = \begin{bmatrix} -1.5590e-001 \\ 3.6630e-002 \\ -1.0356e-015 \end{bmatrix}$$

The first two values are the exact deviation values of the faulty elements R9 and R37 and the R39 is faulty-free.

6. SUMMARY

Fault verification methods applied to linear analog circuits required combinatorial searches of suspected faulty components [4] which made them computationally expensive. In this paper a new fault verification method is proposed based on the modified nodal analysis and identification of the ambiguity groups in the test equation in order to diagnose the multiple faults in the linear analog circuits. Test equation is constructed using the measured circuit responses and the inverse of the coefficient matrix. A developed approach to identify ambiguity groups in the test verification equation is based on the QR factorization techniques. This yields a numerically efficient search for the sets of candidate faulty components. Candidate faulty components are verified by checking independence of the corresponding columns of the original test matrix. Faulty components can be identified in the number of operations $O(n^3)$ rather than $O\left(\begin{pmatrix} n \\ f \end{pmatrix}\right)$ required for combinatorial searches in existing fault verification methods, which is a significant improvement in the computational efficiency. Finally faulty parameters are evaluated through the structural incident signal matrix. An example circuit is provided to illustrate the proposed method.

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