# Averaging Correlation for C/A Code Acquisition and Tracking in Frequency Domain

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***Abstract*:**

This paper describes a new software method called "averaging correlation" to do block level GPS C/A code acquisition and tracking in frequency domain with smaller FFT blocks. The method retains the signal to noise ratio observed in more expensive correlators. Frequency domain software processing will become more feasible and popular if this method is implemented in civil GPS receivers.

## 1 Introduction

Global Positioning System (GPS) has been found useful in most civil surveying and navigation applications, exceeding its original purpose for military service. Civil receivers through out the world are capable of using Standard Positioning Service (SPS) signal, while only authorized users can have Precise Positioning Service (PPS).

Generally speaking, GPS signal format is known as binary shift keying direct sequence spread spectrum [1], [2]. The spread spectrum code designed for civil users is named as Coarse Acquisition (C/A) code, which is on the carrier frequency L1, 1575.42 Hz. Receivers need to acquire C/A code in order to do the Pseudo Range measurements.

In GPS, each of the transmitters (satellites) uses a unique spreading sequence, and the receiver can demodulate any transmitted signal by synchronizing a locally generated PN sequence with the same PN sequence used to generate transmitted signal, taking into account the propagation time delay [3]. The process in which incoming signal is correlated with a local C/A code is called signal acquisition.

Presently the bandwidth of final stage of IF filtering in GPS civil receivers varies from 2 MHz to 20 MHz [1]. In popular low end receivers where 2 MHz bandwidth is preferred, sampling rate is approximately 5 MHz [1]. All data used in this paper is based on this sampling rate.

The process of acquiring satellite signals involves multiple steps. Assuming the code to acquire is already known, simultaneous searches of frequency and code offset are required. This 2-D search can be realized by computing the correlation function of signal and local code. Software radio and FFT algorithm has made advances in receiver design [4], [5].

## 2 Base Band Signal Processing

The received GPS signal is BPSK modulated and a non-coherent demodulation is applied here to process it.

Basically, there are at least three unknown variables left to search, code, code phase, which is known as offset between received code and local code, and IF intermediate frequency (the down converted carrier). However, it is difficult to search with 3 parameters [5]. The code being acquired may be obtained based on previous knowledge so that only the code shift and frequency offsets are considered here in the base band signal processing. It should be noticed that in case of "cold start", even the code type is not available so that the search has to include this parameter [5].

Although the intermediate frequency can be known, it won't be exactly as it is supposed to be, for instance 1.25 MHz, mainly because of Doppler effect. In order to completely remove the IF, search within certain range for frequency with proper step size is necessary for correct signal demodulation. IF residues in the received signal will cause loss of the signal energy [6]. On the other hand, if IF is properly removed, the correlation function will not suffer a reduction of correlation peak height. Thus the search in frequency domain can be realized in this way: try to demodulate the signal with a possible IF, and then check if the correlation function shows the least loss in signal energy at this frequency. The best correlation function corresponds to the highest correlation peak and implies the corresponding frequency is nearest to the true IF.

Since the correlation of signal and local C/A code can also be considered as the search of the relative shift between the local C/A code and the received signal, which is called code offset, this actually becomes a 2-D search for both frequency and the code offset and the navigation data in the signal can be acquired.

After acquisition, tracking loops will be started to do the pseudo range measurements. The main function of the loops is to find out the exact offset between received signal and local C/A code. In most existing receivers, the core component of a tracking loop is named as discriminator [1], which is actually doing sequential correlation computation to find out the offset [1] [7].

Although the job of correlators can also be accomplished in frequency domain, most of them are traditionally sequential and in time domain. Time domain correlators are easy to implement in time domain, since only addition and multiplication operations are needed. On the other hand, a complete search will cost a significant amount of time. Another choice is to do acquisition and tracking in frequency domain, for example, using the Fast Fourier Transform (FFT) converting the GPS signal into the frequency domain [3]. This Fast Fourier Transform (FFT) algorithm is believed to be computationally efficient [4].

However, direct frequency domain acquisition and tracking of the C/A code needs 5000 point FFT processing block, which significantly increases the cost of most civil receiver producers. A new technique will be introduced in the next section to do the frequency domain correlation with much cheaper FFT blocks, which can be realized in more popular hardware.

## 3 The Averaging Correlation Method in Frequency Domain

### 3.1 Frequency Domain Processing

As discussed in the last section, one way to compute correlation in frequency domain is to use 5000 point FFT.

According to the theory of Fourier Transform, the correlation of two signals in time domain can also be expressed in frequency domain, as follows:

$$\int_{-\infty}^{+\infty} x(t)h(t-t)\,dt \Leftrightarrow X(f)H^*(f) \qquad (1)$$

In discrete cases, when both incoming signal and the local code are in the form of 5000 samples per millisecond, the correlation may be obtained by using circular convolution. Since the code will repeat each millisecond and the noise can be considered as random, this circular correlation should be able to accomplish the same job.

$$\sum_{i=1}^{5000} x(i)h(k-i) \Leftrightarrow X(l)H^*(l) = iFFT\,(FFT\,(x(i)) * conj\,(FFT\,(h(i)))) \qquad (2)$$

This frequency domain circular correlation was simulated in MATLAB, using piece of the received GPS signal and an up sampled C/A code, as shown in Fig.1. The corresponding correlation function is demonstrated in Fig.2. As discussed in the last section, the 2-D search for code offset relative to signal and down converted carrier frequency must be performed and the results can be seen in Fig.3.

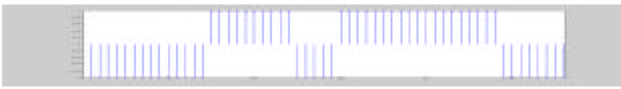


Fig.1 C/A Code Up Sampled (Part)

Unfortunately, 5000 points FFT is to implement in hardware, which might be the reason why frequency domain processor is not popular in present GPS receivers.

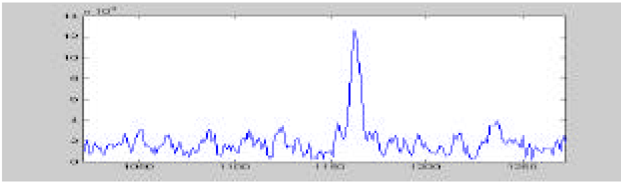


Fig.2 5000 Point Correlation Result (Part Around Peak)

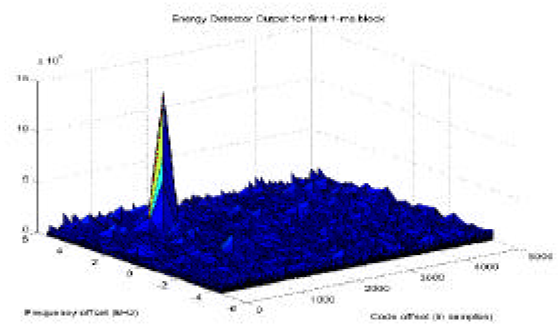


Fig.3 2-D search in code offset and frequency offset

However, there have been cheaper FFT blocks for 1024 points. The original C/A code chip rate is 1023 per millisecond but the signal received is up sampled to be 5000 per millisecond so that receivers have to up sample local code up to this rate. If the signal can be recovered back to the same rate as C/A code, the computation can thus be implemented in the 1024 FFT blocks. How to use the 1024 FFT blocks to do the software signal processing in frequency domain is the basic idea of this design.

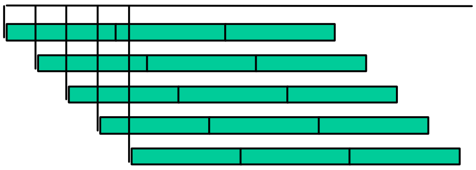
### 3.2 Averaging Up Sampled Signal

In order to use cheaper FFT blocks, the new method called “averaging correlation” is introduced here.

The received signal needs to be recovered back to 1023 chips per millisecond by making consecutive 4 or 5 points averaged into one chip. The averaged chip is similar to a chip of C/A code. Since the signal cannot be observed in forms of square waves, there is not enough information to determine which 4 or 5 points should be grouped together and averaged. Unless there can be approach to know the relative position of the samples and the chips. Although this relative position cannot be exactly found, there can be rough estimates using several time averaging searches. Among the first consecutive 5 samples, one of them must be the first sample of a chip, since no chip contains more than 5 samples. And if this sample is regarded as the beginning position of the first chip, the relative position of the 1023 chips and all the samples can be determined and the chips can be recovered completely. Although that beginning point would not be the exact starting position in normal cases, it is a fairly good approximation. Thus the problem becomes searching for the beginning sample within the 5 ones in the front of the samples, as shown in Fig.4. Beginning with the right point, the averaged sequence of chips is nearest to the original one and the correlation peak of this sequence and the local C/A code is higher than others. So 5 sequences beginning with the first 5 samples respectively are tried and one of the 5 would be chosen as the starting point of chips. However, all the 5 sequences have their physical meanings. The correlation of C/A code and these 5 sequences respectively would produce 5*1023 values.

Actually the time location of each one of these values represents a particular relative shift in time domain between received signal and local code. Obviously the shift represented by the largest peak is nearest to the real time shift between signal and code. Determining this shift is the aim of tracking and it will be discussed in section 3.3.

*5000 samples of the signal (or called 5000 bits)*



Each sequence has 1023 chips

Fig.4 The 5 Sequences of Averaged Steps.

### 3.3 Acquisition with Cheaper FFT

With the 5 averaged sequences and 1023 chip C/A code, the correlation computation needs to be done 5 times. Notice that 5 times 1023 FFT is still cheaper than 5000 FFT, no matter in hardware or just software simulation in MATLAB.

In the case this method is only applied to rough acquisition, the correlation of one or two out of these 5 sequences with C/A code can already locate the peak within 1/1023 millisecond, as shown in Fig.5 and Fig.6.

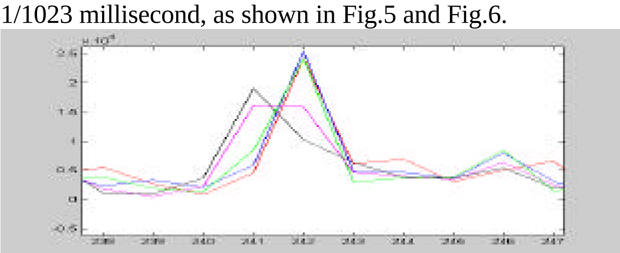


Fig.5 Correlation for All the 5 Sequences And C/A Code

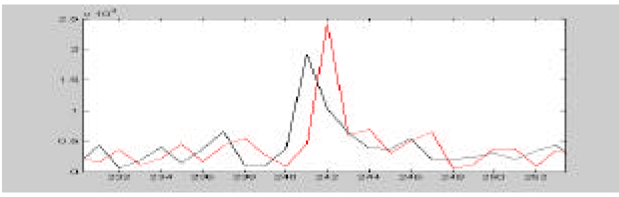


Fig.6 Correlation for 2 of the 5 Sequences And C/A Code

A good point of this so called “averaging correlation” is that it is not going to cause loss of signal energy compared with correlation by 5000 FFT. It is difficult to define the signal to noise ratio for the correlation function of signal with noise and the local C/A code, because the C/A code auto correlation function itself has side lobes and the side lobes of signal-C/A correlation can also comprise the peak locating in case of large noise. However, the peak position that indicates the time shift is the only information needed. So for the correlation functions the traditional definition of signal to noise ratio is not used. Instead, the strength ratio of peak to second largest peak is introduced to determine the signal strength over the noise.

If the peak value found at a certain position of the received signal over the second largest peak is always greater than a certain level, the signal can be regarded as strong over the noise.

This ratio actually indicates whether or not the relative shift can be credibly found. By looking at this ratio from the 5000 FFT correlation function and the averaging correlation function, the performance of these two methods can be easily compared.

When the correlation result of one sequence has the largest peak value among all 5 averaged sequences, this one is considered to be the best recovery of the original chips within the received signal, as discussed in Section 3.1. The ratio mentioned above is computed for this sequence and compared with the ratio obtained from 5000 FFT correlation function. This comparison is done throughout 200 consecutive milliseconds of received GPS signal, statistical results shown in Fig.7 and 8 indicate that the ratio performance of the averaging method won’t be worse than 5000 FFT correlation, if not better.

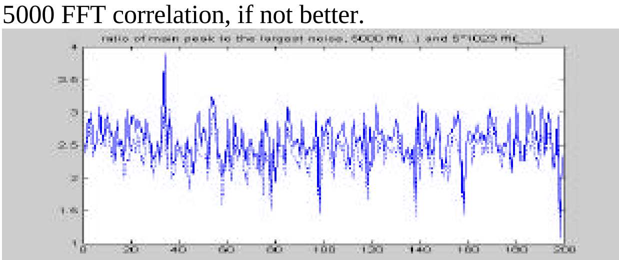


Fig.7 Peak to the Second Largest Ratio Comparison of the Two Methods (Over 200 Millisecond Data)

In Fig.7, the dashed line stands for the ratio of peak to second largest peak obtained from the correlation function by 5000 point FFT over 200 millisecond data. The mean ratio is 2.3675 with the variance 0.0915, as illustrated in Fig.8. The continuous line stands for the similar ratio from correlation function obtained by averaging method also over 200 milliseconds. And the mean is 2.5776 with variance 0.1144, as shown in Fig.8. Its mean values it can be found that averaging method seems a little bit more stable than the 5000 FFT correlation. Fig.8 indicates the histogram of this two ratio values through out the 200 milliseconds of data.

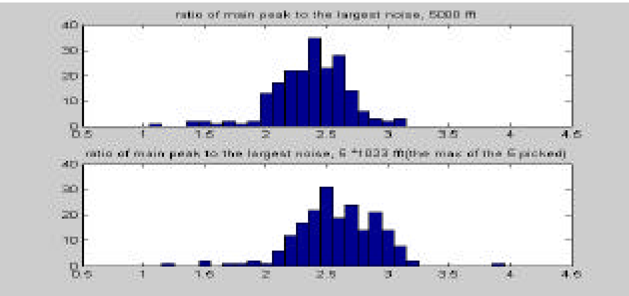



Fig.8 Peak to the Second largest Ratio Comparison

In the conclusion, the averaging correlation can work as well as the 5000 FFT method in signal acquisition.

### 3.4 Tracking and Precise Peak Location

Traditionally after the code is acquired from the signal, tracking loop will be started to precisely find the relative shift [1]. However, in the proposed method there is no need to divide acquisition and tracking into two different procedures in frequency domain process. As mentioned before, 5 sequences of chips will be obtained from the correlation computation. Each chip out of these 5*1023 ones stands for a unique time shift value. The chip of the largest peak indicates that the time shift is nearest to the real relative shift of signal and local code. The shift values represented by these chips and the corresponding correlation function values can be combined together as a whole discrete correlation function, which is quite similar to the correlation function obtained with 5000 FFT method. Knowing the shape of the ideal continuous correlation function, this discrete function can be fitted into certain kind of curves to recover the continuous one. In that case the true peak of the ideal continuous function can be located, which is supposed to represent the exact relative shift, the final purpose of tracking.

Obviously, the ideal auto correlation of C/A code should be a certain symmetric triangle with side lobes according to C/A code autocorrelation properties. However, the correlation of received signal and local code is not perfect even if there is no thermal noise due to the distortions cast onto the signal all through the propagation. As a result, the peak of the correlation function has a curve at the top instead of a sharp peak. It might not be symmetric due to multipath [1] [8]. As mentioned above, different curves can be fitted into the discrete sequences to simulate the real shape of the correlation function. The easiest way is to assume the shape is still a symmetric triangle to find out the exact shift, within certain precision. Another possible solution is to fit polynomial curves at the top of the discrete correlation function as a simulation for the peak shape. The precise peak position can thus be found. It would be hard to say what kind of curve is better since the shape of the correlation function is not certain. However it is predictable that if there are no heavy multipath effects, the correlation peak may still appear symmetric and the triangular fit may works well.

## 4 Future Works and Conclusion

This developed averaging correlation method explains how to use standard 1024 FFT core to do acquisition for 5000 sample block of received signal values.

Although there is more research needed to compare this developed averaging correlation method with other acquisition and tracking solutions in both time domain and frequency domain, it must noted that this method will be beneficial to commercial GPS receivers. It can lower the cost of these receivers and at the same time improve the performance, which may make the software frequency domain processing more popular in the civil receivers. The implementation of this method is being developed. A similar method may also be considered for PPS signal processing, which can help to realize the direct acquisition of P-code in civil receivers.

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