SENSITIVITY BASED TESTING OF NONLINEAR CIRCUITS

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ABSTRACT

The paper presents a method designed to test nonlinear circuits using sensitivity approach. Several excitation levels and signal frequencies are used to improve system testability. This method is illustrated using piecewise linear models, however, it can be generalized to handle other nonlinear characteristics. QR algorithm is used to select test points and ensure numerical stability. In case of sufficient number of measurement points a linear, fault verification technique can be also used.

I. INTRODUCTION

Testing and calibration of electronic devices and circuits must take into consideration all practical aspects of both the test itself and the post test operations. The number of nodes (pins) available for the test is usually limited, test costs increase with the number of selected test points, and measurement errors limit the accuracy of the element identification. Post test operations include computer–aided solution of the element identification problem or the numerical evaluation of predicted variations of the circuit response.

Using the design of experiments approach or other optimization techniques, one can maximize the accuracy with a given limitation on the number of tests. One of such practically acceptable optimization techniques [1] is based on the QR algorithm, suitable for efficient test point selection in a large circuit.

Most of the practical systems are nonlinear and can be handled through an appropriate linearization of system equations. For large nonlinearities, one can diagnose the system by using piecewise linear approach and test it under small signal sinewave excitations. Recent papers [2] and [3] discuss the problem of nonlinear fault diagnosis through a linear fault verification process. In both papers, the number of test nodes have to be greater than the number of faults and a number of combinations have to be checked in order to locate and verify faults. Nonlinear elements could be identified through their values at the system operating point.

The main objective of this paper is to present a method, which can be used for fault diagnosis of nonlinear circuits with small variations of circuit parameters. Characteristics of nonlinear elements can be identified either directly or through a piecewise linear approach with different DC excitation levels. An effect of nonlinearities in network elements on the rank of the system sensitivity matrix is studied. It is shown that the presence of nonlinear elements in the system facilitates fault diagnosis problem by providing an additional information at the measurement points.

II. FAULT DIAGNOSIS EQUATIONS

Let us consider a nonlinear network described by the nodal equations. Formulate the system function:

$$f(\mathbf{v}_n,\mathbf{p}) = \mathbf{A}\,\mathbf{i}_b(\mathbf{A}^T\,\mathbf{v}_n) - \mathbf{j}_n\,, \tag{1}$$

where $\mathbf{i}_b(\mathbf{A}^T\mathbf{v}_n) = \mathbf{i}_b(\mathbf{v}_b)$ represents the element equations (branch currents), $\mathbf{v}_n$ and $\mathbf{v}_b$ represent nodal and branch voltages respectively, $\mathbf{A}$ is the incidence matrix and $\mathbf{j}_n$ represents the nodal current excitations.

For the nominal values of the system parameters $\mathbf{p}_0$ at a given DC operating point we have

$$f(\mathbf{v}_n^*,\mathbf{p}_0) = \mathbf{0}\,. \tag{2}$$

In our analysis we assume that the system parameters $\mathbf{p}$ are close to their nominal values. The purpose of the fault diagnosis is to find deviations $\Delta\mathbf{p}=\mathbf{p}-\mathbf{p}_0$, which characterize changes in the element equations. Linear elements are described through their admittances, therefore only one parameter is required to identify each linear element. Nonlinear elements have characteristics described through several parameters $\mathbf{p}_i$ (e.g. $i_b = p_0 \exp(p_1 v_b) + p_2$), so one nonlinear element may require identification of more than one value in order to obtain its characteristic.

Let us assume that the excitation currents do not depend on the network parameters. The sensitivity matrix at the nominal point can be determined by taking derivative of (1)

$$\mathbf{A}\,\left(\frac{\partial \mathbf{i}_b}{\partial \mathbf{v}_b}\,\mathbf{A}^T\,\frac{\partial \mathbf{v}_n}{\partial \mathbf{p}} + \frac{\partial \mathbf{i}_b}{\partial \mathbf{p}}\right) = \mathbf{Y}_n(\mathbf{v}_n,\mathbf{p})\,\frac{\partial \mathbf{v}_n}{\partial \mathbf{p}} + \mathbf{A}\,\frac{\partial \mathbf{i}_b}{\partial \mathbf{p}} = \mathbf{0}\,. \tag{3}$$

Solution of this equation w.r.t. $\frac{\partial \mathbf{v}_n}{\partial \mathbf{p}}$ yields

$$\frac{\partial \mathbf{v}_n}{\partial \mathbf{p}} = -\,\mathbf{Y}_n^{-1}\,\mathbf{A}\,\frac{\partial \mathbf{i}_b}{\partial \mathbf{p}}\,. \tag{4}$$

Notice that $\frac{\partial \mathbf{i}_b}{\partial \mathbf{p}}$ is calculated at the nominal point and is a function of both voltage and parameter values. To obtain $\frac{\partial \mathbf{v}_n}{\partial \mathbf{p}}$, an analysis similar to the adjoint network approach can be conducted. The admittance matrix Y_n have to be updated for each new operating point since it is a function of the nominal solution. Changing operating points of the system is equivalent to setting the element admittances to new values.

Fault diagnosis of the nonlinear equations is conducted as follows:

1. Set DC excitation to a specific value and solve nonlinear equations to obtain the nominal operating point.
2. Use a linearized model for the purpose of small signal analysis and calculate voltage responses at the measurement points.
3. Use the difference between the measured and the calculated voltages to formulate the fault diagnosis equations

$$S\frac{\Delta p}{p} = \frac{\Delta v}{v}, \tag{5}$$

where $s_{ij} \epsilon S$ is as follows:

$$s_{ij} = \frac{\partial v_i}{\partial p_j}\frac{p_j}{v_i}. \tag{6}$$

4. If additional fault diagnosis equations are needed go to Step 1, otherwise solve (5) to obtain deviations $\Delta \mathbf{p}$.

A special case of the discussed method is obtained when the nonlinear elements have piecewise linear characteristics. This case may serve well as an illustration for the proposed approach, and some general observations are easier to be discussed while illustrated in the piecewise linear network.

III. PIECEWISE LINEAR FAULT DIAGNOSIS

In the case of piecewise linear characteristics the differentiations of element equation needed in (4) produces several discrete values of the element admittances and the associated current sources. Note that the nodal approach limits the type of nonlinear resistors to the voltage controlled characteristics. This limitation can be easily removed by using the modified nodal approach. The nodal approach has been chosen only for the purpose of simplicity.

Test equations

Consider for example an element whose v–i characteristic is shown in Fig. 1.

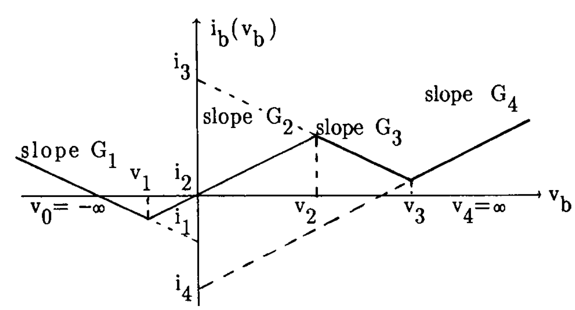


Fig. 1 Characteristic of a piecewise linear element.

In each interval of branch voltages, branch current is described by a function

$$i_b(v_b) = i_c + G_c v_b \qquad \text{for } v_{c-1} \le v_b \le v_c, \tag{7}$$

so its derivatives are

$$\frac{\partial i_b}{\partial v_b} = G_c, \qquad \frac{\partial i_b}{\partial i_c} = 1 \quad \text{and} \quad \frac{\partial i_b}{\partial G_c} = v_b. \tag{8}$$

If the element is located between nodes i and j, then $v_b = v_i - v_j$. Its derivative $\frac{\partial v_n}{\partial G_c}$ in (4) will be obtained in the similar way to the sensitivities in the linear network, in which Y_n^{-1} is calculated with values G_c substituted for each nonlinear element. We have

$$\frac{\partial v_n}{\partial G_c} = (v_i^a - v_j^a)(v_i - v_j) \qquad \text{and} \qquad \frac{\partial v_n}{\partial i_c} = -(v_i^a - v_j^a). \tag{9}$$

System testability

To study properties of the sensitivity matrix we represent a measured voltage v_q in the symbolic form

$$v_q = \frac{h_0 + \sum_i h_i G_i + \sum_{i<j}\sum h_{ij} G_i G_j + \sum_i \sum_{<j<k}\sum h_{ijk} G_i G_j G_k + \cdots}{b_0 + \sum_i b_i G_i + \sum_i \sum_{<j} b_{ij} G_i G_j + \sum_i \sum_{<j<k}\sum b_{ijk} G_i G_j G_k + \cdots} \tag{10}$$

where each $h_t(h_0, h_i, h_{ij}, \ldots, h_{ijk})$ is a linear combination of $(\eta+1)$ current sources if there are η piecewise linear elements in the circuit.

$$h_t = a_0 J_s + a_1 i_{c1} + \cdots + a_s i_{cs} + \cdots + a_\eta i_{c\eta}. \tag{11}$$

Coefficients a_s and b_t depend on the values of linear elements, $i_{c1}, \ldots, i_{c\eta}$ are the cut–off sources from piecewise characteristics, and J_s is the current excitation.

The output voltage v_q will be different when the nonlinear element values (admittance or cut–off current) change. Assume that the number of voltage measurements is m, the upper bound on the number of independent measurements can be estimated by using the result of [5]

$$R_{max} = T - 1 + \sum_{q=1}^{m} T_q, \tag{12}$$

where T and T_q are the number of nonzero coefficients in the denominator and numerator of v_q respectively. From (11) and (12), we get

$$T \le \left(\sum_{i=1}^{\eta} c_\eta^i\right) = 2^\eta, \qquad \text{and} \qquad T_q \le \eta\, 2^\eta, \tag{13}$$

so

$$R_{max} = 2^\eta - 1 + m * \eta * 2^\eta. \tag{14}$$

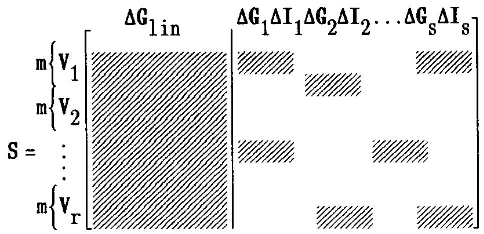
Testability of the circuit can be estimated by the rank of the sensitivity matrix S of the system equations. The effect of nonlinearities in the nonlinear network on the rank of S is similar to that of multifrequency measurements in the linear reactive network. The upper bound on the rank of the sensitivity matrix of the system can be approximated by R_{max} in (14). In the piecewise linear approach, not all values will be attained independently, therefore the rank of the sensitivity matrix is lower than (14).

Sensitivity matrix

In order to obtain additional equations, voltage measurements are taken at the same test nodes under different DC excitation levels and different frequencies. Since the voltage measurements are taken at different regions of the piecewise characteristics, this method is called the multiregion measurements test.

Let $\Delta \underset{\sim}{G}_{lin}$ be the vector of deviations of linear elements from their nominal values and $\Delta \underset{\sim}{G}_\ell$, $\Delta \underset{\sim}{I}_\ell$ the vectors of deviations in nonlinear segments description (changes in i_α and G_α in each segment for each nonlinear element). Change of the region causes the number of unknowns to increase as new values for ΔG_ℓ and ΔI_ℓ have to be added. However, if the number of frequency dependent measurements at each region is larger than the number of added variables, then we gain additional system information. In such case we will be able to evaluate all the unknowns.

Let r be the number of regions and s be the number of segments. The sensitivity matrix has the following pattern



(15)

Example

Consider the nonlinear circuit shown in Fig. 3. The equivalent circuit is shown in Fig. 4 and the characteristics of the piecewise linear elements g_1 and g_2 are given in Fig. 5. Voltage measurements are taken at the nodes 1,2 and 3.

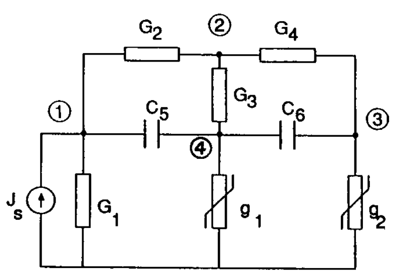


Fig. 3 An nonlinear circuit

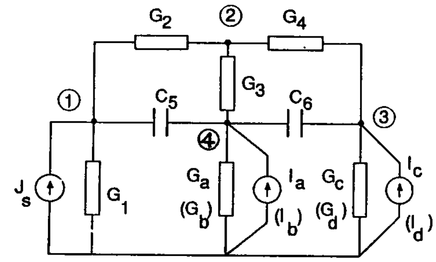


Fig. 4 The equivalent circuit

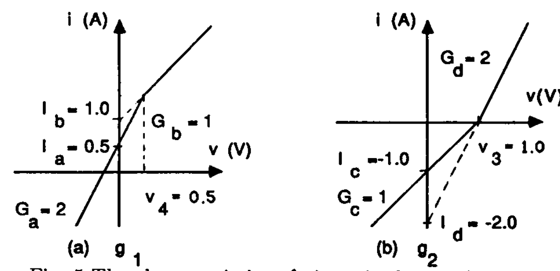


Fig. 5 The characteristics of piecewise linear elements.

Each piecewise linear element has two segments and the space is divided into four regions as follows:

r_1 (segment A and C): $v_3 < 1.0$ V and $v_4 < 0.5$ V;
r_2 (segment B and D): $v_3 > 1.0$ V and $v_4 > 0.5$ V;
r_3 (segment B and C): $v_3 < 1.0$ V and $v_4 > 0.5$ V;
r_4 (segment A and D): $v_3 > 1.0$ V and $v_4 < 0.5$ V.

The operating point can be moved from one region to another by changing the level of the DC current excitation (r_1: J_s=-2.0 A, r_2: J_s=3.0 A and r_3: J_s=15.0 A). At each region , a small AC signal is applied at two test frequencies: f_1=0.075 Hz and f_2=0.01 Hz.

Assume that the deviations of element values D ($\Delta p/p$) are small (0.5% – 23%). The QR algorithm was run on the obtained sensitivity matrix (15) to select 14 test points. Then the element deviations were computed by solving (5). The results of computer simulation are listed in Table 1.

Table. 1 Computer Results

element		computed D_c	actual D_a	$\Delta D = D_c - D_a$
1	G1	-0.00973	-0.010	0.00027
2	G2	0.00447	0.005	-0.00053
3	G3	-0.02129	-0.020	-0.00205
4	G4	-0.01070	-0.010	-0.00069
5	C5	0.00784	0.010	-0.00216
6	C6	-0.02205	-0.020	-0.00205
7	Ga	-0.05150	-0.050	-0.00150
8	Ia	0.22660	0.226	0.00060
9	Gb	0.01986	0.020	-0.00014
10	Ib	-0.01973	-0.020	-0.00150
11	Gc	-0.01069	-0.010	-0.00069
12	Ic	-0.01070	-0.010	-0.00070
13	Gd	0.00444	0.005	-0.00056
14	Id	0.00435	0.005	-0.00065

IV. LINEAR VERIFICATION METHOD

In a special case when the number of measurement points exceeds the number of faults in the circuit ($m > f$) a linear verification technique similar to [2] can be used. Here we show that in order to identify faults in nonlinear elements described by a piecewise linear characteristic we do not have to set up additional test points. Both the slope and the current cut–off values can be evaluated with no additional requirements for the verification technique.

Let ℓ denote the region (linear combination of segments) in which the network N_ℓ operates. The nodal equations of N_ℓ obtained for the nominal element values have the form

$$Y_\ell V_\ell = I + I_\ell . \tag{16}$$

If the network N_ℓ is perturbed to $(N_\ell + \Delta N_\ell)$ with the same excitations we obtain

$$(Y_\ell + \Delta Y_\ell)(V_\ell + \Delta V_\ell) = I + I_\ell + \Delta I_\ell . \tag{17}$$

Subtracting (16) from (17) yields

$$Y_\ell \Delta V_\ell + \Delta Y_\ell V'_\ell = \Delta I_\ell , \tag{18}$$

where

$$V'_\ell = V_\ell + \Delta V_\ell . \tag{19}$$

Let

$$Y_\ell \Delta V_\ell = \Delta I , \tag{20}$$

(18) becomes

$$\Delta I = \Delta I_\ell - \Delta Y_\ell V'_\ell . \tag{21}$$

ΔI_ℓ represents the changes of the current sources due to the changes of nonlinear elements only and $\Delta Y_\ell V'_\ell$ represents the changes of the nodal currents due to the admittance changes of both linear and nonlinear elements. Assume that only f elements of ΔI are nonzero (i.e. $\Delta I = \Delta I^f$) and we measure the voltage response at m nodes ($m \geq f$). From (20) we have

$$\Delta V^m_\ell = Z_{mf} \Delta I^f . \tag{22}$$

where

$$Z_{mf} = (Y_\ell^{-1})_{mf} . \tag{23}$$

At this point the fault diagnosis equations (22) can be solved using the voltage measurements V^m and the verify and test approach as in [5]. As a result of the fault diagnosis we obtain ΔI^f. Using ΔI^f in (20) for ΔI we can evaluate ΔV_ℓ and therefore V'_ℓ.

The faulty elements in the region ℓ will be identified if ΔY_ℓ and ΔI_ℓ are evaluated. To obtain additional equations without changing the test nodes and the excitation nodes, additional measurements are taken at the same test nodes under a different DC excitation level. Values of the applied excitations have to be chosen to keep the network operating in the same region (say the region ℓ in this case) so that ΔY_ℓ and ΔI_ℓ will not change. The additional equations are

$$\Delta I_{(2)} = \Delta I_\ell - \Delta Y_\ell V'_{\ell(2)} . \tag{24}$$

Combining (21) and (24) we have two sets of equations which can be used to find ΔY_ℓ and ΔI_ℓ

Solution of (22) gives us changes in the nodal current while (21) deals with branch currents and admittances. Other approaches directed towards the branch fault diagnosis rather than the nodal fault diagnosis are possible (see [6]).

V. CONCLUSIONS

The paper presents a sensitivity based method to test nonlinear circuits with small variations of element parameters. The basic notations and results discussed in the paper refer to the piecewise linear description of nonlinear elements, where we assume that each nonlinear element can be approximated by a specified number of linear segments. The purpose of this representation is to simplify the discussion on the fault diagnosis.

The method can be generalized to the case where nonlinear elements are described by any function on the voltage–current plane. If such a function is uniquely characterized by a set of parameters, then this nonlinear element is diagnosed when parameters are identified. In the particular case of a piecewise linear characteristic we need to identify two parameters per segment namely the slope and the current cut–off value.

We have illustrated this approach with an example in which all linear and nonlinear elements were identified. In a case when the number of measurement points exceeds the number of faults a linear verification technique can be used.

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VI. REFERENCES

[1] G. N. Stenbakken, and T. M. Souders, "Test point selection and testability measures via QR factorization of linear models", Proc. 29th Midwest Symp. on Circuits and Syst., (Lincoln, Nebraska), 1986.

[2] Q. Huang and R. W. Liu, "Fault diagnosis of piecewise linear system", Proc. IEEE Int. Symp. Circuits and Systems (Philadelphia, PA), pp. 884–887, 1987.

[3] M. E. Zaghloul and D. Gobovic, "Single–fault diagnosis of nonlinear resistive networks", IEE Proceedings, Vol. 134, Pt. G, No. 1, 1987.

[4] R. S. Berkowitz, "Conditions for network element value solvability." IRE Trans. Circuit Theory, vol. CT–9, pp 24–29, 1962.

[5] J. A. Starzyk and J. W. Bandler, "Nodal approach to multiple–fault location in analog circuits", Proc. IEEE Int. Symp. Circuits and Systems, (Rome, Italy), pp. 1136–1139, 1982.

[6] C. S. Lin, Z. F. Huang and R. W. Liu, "Topological conditions for single branch–fault", IEEE Trans. Circuits and Systems, vol. CAS–30, pp.376–381, 1983.