FAULT DIAGNOSIS AND CALIBRATION OF LARGE ANALOG CIRCUITS

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ABSTRACT

This paper presents a new strategy in testing of large, analog circuits. A tested circuit is partitioned at nodes were the voltage measurements are taken. Test equations are formulated on the basis of Kirchhoff's current law equations at the partition points. This results in the Jacobian matrix with a sparse block structure. The test points selection and the element evaluation can be performed in parallel, reducing computation time and enhancing the test performance.

I. INTRODUCTION

Fault diagnosis and calibration of analog circuits are related. When the changes in elements are small, a relationship between measurement deviations can be expressed through the sensitivity matrix. With a sufficient number of the test points all elements can be evaluated. This allows one to diagnose the faulty circuit or calibrate a working one tuning its elements to the desired settings.

A popular approach used in testing and calibration is to excite a tested circuit with sinewave excitations and measure deviations of its steady–state voltage responses from their nominal values. By changing frequency, excitation and measurement points, a sufficient number of independent measurements can be produced. As a result an independent system of test equations based on the sensitivity matrix can be formulated. Discussion about the effect of the network topology, type of its elements and test points on the rank of the sensitivity matrix can be found in several papers addressing problems of network testability [1], [2] and [3].

Analyzing the sensitivity matrix with the help of the QR algorithm, one can select a sub–optimum set of test points, to evaluate circuit elements with minimum computational effort and high numerical accuracy as reported in [4]. Using the same technique, an effect of measurement errors and element tolerances can be studied with sufficient statistical confidence.

The main drawback of the sensitivity based method is its relatively high computational cost. Each transfer function is sensitive to variations of every network parameter, which causes the sensitivity matrix to be a dense matrix. This makes numerical calculations to be expensive in the case of large networks. In addition, higher requirements for the computer memory, limits the size of a network that can be effectively handled.

The main purpose of this paper is to present a method without these deficiencies. A network is partitioned into subnetworks and the voltage measurements at the partition points are used as discussed in [5]. The system test equations are formed based on Kirchhoff's current law (KCL) equations at the test points. As a result the obtained Jacobian matrix is a sparse matrix. The test points selection can be implemented by performing the QR factorization within each subnetwork in parallel. Both the parameter deviations and the testability of each element can be obtained locally. Off–line and on–line computations are largely reduced.

II. NETWORK DECOMPOSITION

In this section the basic equations of the proposed method will be outlined. System test equations for the network partitioned at the measurement points are formulated and the Jacobian matrix is derived.

Network Decomposition

Let n+1 be the number of nodes of a network N, m be the number of measurements nodes (or external nodes), i be the number of internal nodes, and e be the number of external current excitations.

The nodal voltage equation for the network is

$$\underset{\sim}{Y}_n \underset{\sim}{V}_n = \underset{\sim}{I}_n. \qquad (1)$$

Label the external nodes first and then the internal nodes, in which case the nodal admittance matrix is

$$\underset{\sim}{Y}_n = \begin{bmatrix} \underset{\sim}{Y}_{mm} & \underset{\sim}{Y}_{mi} \\ \underset{\sim}{Y}_{mi} & \underset{\sim}{Y}_{ii} \end{bmatrix}. \qquad (2)$$

The corresponding external (shorted) admittance matrix can be written as follows:

$$\underset{\sim}{Y}_m = \underset{\sim}{Y}_{mm} - \underset{\sim}{Y}_{mi}\, \underset{\sim}{Y}_{ii}^{-1}\, \underset{\sim}{Y}_{im} \quad . \qquad (3)$$

The network N can be divided into k subnetworks connected by the nodes of decomposition as shown in Fig. 1. There should be no mutual coupling between any two subnetworks and the nodes of decomposition should be chosen from the set where voltage measurements are performed.

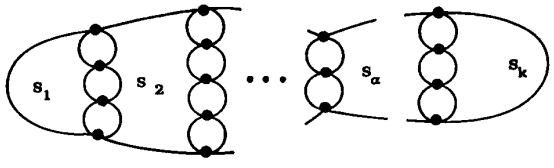

Fig. 1 Subnetworks of N

If the subnetwork S_α has m_α measurement nodes and i_α internal nodes, the corresponding external (shorted) admittance matrix can be written as follows:

$$\underset{\sim}{Y}_{m_\alpha} = \underset{\sim}{Y}_{m_\alpha m_\alpha} - \underset{\sim}{Y}_{m_\alpha i_\alpha} \underset{\sim}{Y}_{i_\alpha i_\alpha}^{-1} \underset{\sim}{Y}_{i_\alpha m_\alpha}, \quad \text{for } \alpha=1,2,..k, \quad (4)$$

Equations (4) can be computed in parallel and the external admittance matrix $\underset{\sim}{Y}_m$ can be obtained by adding different $\underset{\sim}{Y}_{m_\alpha}$ in the block diagonal form as shown

$$\underset{\sim}{Y}_m = \begin{bmatrix} \boxed{\underset{\sim}{Y}_{m_1}} & & & \\ & \boxed{\underset{\sim}{Y}_{m_2}} & & \\ & & \boxed{\underset{\sim}{Y}_{m_\alpha}} & \\ & & & \boxed{\underset{\sim}{Y}_{m_k}} \end{bmatrix} . \quad (5)$$

System Test Functions and Test Equations

According to Kirchhoff's current law (KCL), the nodal equations at the external nodes are

$$\underset{\sim}{Y}_m \underset{\sim}{V}_m = \underset{\sim}{I}_m , \quad (6)$$

where $\underset{\sim}{Y}_m$ is the (mxm) external admittance matrix, $\underset{\sim}{V}_m$ is the (mxe) external voltage matrix which can be obtained from measurements, and $\underset{\sim}{I}_m$ is the (mxe) external current matrix, which contains excitations applied at the external nodes.

Let $\underset{\sim}{X}$ be a vector of p elements in the network. It is known that the external admittance matrix $\underset{\sim}{Y}_m$ is a function of element values

$$\underset{\sim}{Y}_m = \underset{\sim}{Y}_m (\underset{\sim}{X}) . \quad (7)$$

Define sums of currents at external nodes as the system test functions

$$\underset{\sim}{F}(\underset{\sim}{X}) = \text{vec} [\underset{\sim}{Y}_m(\underset{\sim}{X}) \underset{\sim}{V}_m - \underset{\sim}{I}_m] . \quad (8)$$

According to Kirchhoff's current law the sum of currents at any node is equal to zero

$$\underset{\sim}{F}(\underset{\sim}{X}^*) = \underset{\sim}{0} . \quad (9)$$

where $\underset{\sim}{X}^*$ indicates the set of elements in the faulty circuit in which the measurement vector $\underset{\sim}{V}_m$ was obtained. To indicate voltage dependence on $\underset{\sim}{X}^*$ explicitly we may refer to $\underset{\sim}{V}_m$ as $\underset{\sim}{V}_m(\underset{\sim}{X}^*)$.

Assume that the deviations of element values from the nominal values $\underset{\sim}{X}_0$ are small. The Taylor expansion can be used to produce

$$\underset{\sim}{F}(\underset{\sim}{X}^*) \cong \underset{\sim}{F}(\underset{\sim}{X}_0) + \nabla_{\underset{\sim}{X}} \underset{\sim}{F}(\underset{\sim}{X}) \Big|_{\underset{\sim}{X}=\underset{\sim}{X}_0} \Delta\underset{\sim}{X} . \quad (10)$$

Denote $\underset{\sim}{J}$ to be the Jacobian (or sensitivity) matrix

$$\underset{\sim}{J} = \nabla_{\underset{\sim}{X}} \underset{\sim}{F}(\underset{\sim}{X}) . \quad (11)$$

The system of test equations is as follows

$$\underset{\sim}{J} \Delta\underset{\sim}{X} = - \underset{\sim}{F}(\underset{\sim}{X}_0) . \quad (12)$$

The Jacobian matrix consists of the derivatives of all functions w.r.t. each element. Let p be the number of elements in the circuit. With known measurement voltages $\underset{\sim}{V}_m$ and excitation currents $\underset{\sim}{I}_m$ variations in $\underset{\sim}{F}(\underset{\sim}{X})$ w.r.t. x_ℓ depend on variations of $\underset{\sim}{Y}_m(\underset{\sim}{X})$ w.r.t. x_ℓ only, so

$$[\underset{\sim}{J}]_\ell = \frac{\partial \underset{\sim}{F}(\underset{\sim}{X})}{\partial x_\ell} = \{\text{vec}[\frac{\partial \underset{\sim}{Y}_m(\underset{\sim}{X})}{\partial x_\ell} \underset{\sim}{V}_m]\}_\ell , \quad \text{for } \ell=1,2,...,p . \quad (13)$$

III. EVALUATION OF THE JACOBIAN MATRIX

The Jacobian matrix (13) can hardly be evaluated directly, since the symbolic expression for $\underset{\sim}{Y}_m(\underset{\sim}{X})$ is usually unknown. To evaluate the Jacobian, we refer to (3) and use the following derivation.

Form $$\underset{\sim}{Y}_n^{-1} = \begin{bmatrix} \underset{\sim}{Y}_{mm} & \underset{\sim}{Y}_{mi} \\ \underset{\sim}{Y}_{im} & \underset{\sim}{Y}_{ii} \end{bmatrix}^{-1} = \begin{bmatrix} \underset{\sim}{Z}_{mm} & \underset{\sim}{Z}_{mi} \\ \underset{\sim}{Z}_{im} & \underset{\sim}{Z}_{ii} \end{bmatrix} , \quad (14)$$

we have

$$\underset{\sim}{Y}_m = \underset{\sim}{Z}_{mm}^{-1} . \quad (15)$$

For a square matrix $\underset{\sim}{A}$ the following relationship is satisfied

$$\frac{\partial \underset{\sim}{A}^{-1}}{\partial x} = -\underset{\sim}{A}^{-1} \frac{\partial \underset{\sim}{A}}{\partial x} \underset{\sim}{A}^{-1} . \quad (16)$$

After derivation, we obtain

$$\frac{\partial \underset{\sim}{Y}_m}{\partial x} \underset{\sim}{V}_m(\underset{\sim}{X}^*) = -\underset{\sim}{Y}_m \frac{\partial \underset{\sim}{Z}_{mm}}{\partial x} \underset{\sim}{Y}_m \underset{\sim}{V}_m(\underset{\sim}{X}^*) = \underset{\sim}{H} \frac{\partial \underset{\sim}{Y}_n}{\partial x} \underset{\sim}{V}_n(X'), \quad (17)$$

where

$$\underset{\sim}{H} = [\underset{\sim}{U}_{mm} \mid -\underset{\sim}{Y}_{mi} \underset{\sim}{Y}_{ii}^{-1}], \quad (18)$$

and

$$\underset{\sim}{V}_n(X') = \begin{bmatrix} \underset{\sim}{V}_m(\underset{\sim}{X}^*) \\ -\underset{\sim}{Y}_{ii}^{-1} \underset{\sim}{Y}_{im} \underset{\sim}{V}_m(\underset{\sim}{X}^*) \end{bmatrix} = \begin{bmatrix} \underset{\sim}{V}_m(\underset{\sim}{X}^*) \\ \underset{\sim}{V}_i(\underset{\sim}{X}') \end{bmatrix} . \quad (19)$$

The voltage vector includes the external voltages obtained from measurements and internal voltages calculated by (19). Substituting (17) into (13), we obtaine

$$\underset{\sim}{J}_{\ell} = \mathrm{vec}[\underset{\sim}{H} \frac{\partial \underset{\sim}{Y}_n}{\partial x_{\ell}} \underset{\sim}{V}_n(\underset{\sim}{X}')]_{\ell}, \qquad \text{for } \ell=1,2,...,p. \tag{20}$$

It can be shown form (18) that $\underset{\sim}{H}$ has the block structure as follows:

$$\begin{array}{c} m_1\ m_2 \cdots m_{\alpha} \cdots m_k \quad i_1\ i_2 \cdots i_{\alpha} \cdots i_k \\ \left[\begin{array}{ccccccc} \underset{\sim}{U}_1 & & & & \boxed{W_1} & & \\ & \underset{\sim}{U}_2 & & & \boxed{W_2} & & \\ & & \ddots & & & \ddots & \\ & & & \underset{\sim}{U}_{\alpha} & & \boxed{W_{\alpha}} & \\ & & & & \ddots & & \ddots \\ & & & & & & \boxed{W_k} \\ & & & \underset{\sim}{U}_k & & & \end{array}\right] \begin{array}{c} m_1 \\ m_2 \\ \vdots \\ m_{\alpha} \\ \vdots \\ m_k \end{array} \end{array}$$

where

$$\underset{\sim}{W}_{\alpha} = -\underset{\sim}{Y}_{m_{\alpha} i_{\alpha}} \underset{\sim}{Y}^{-1}_{i_{\alpha} i_{\alpha}}, \tag{21}$$

(21) can be computed in parallel within each subnetwork so the computations can be reduced.

Suppose that the element x_{ℓ} is in the subnetwork S_{α} and is incident to nodes i and j. Let f_{qr} be the sum of currents at node q (q=1,2,...,m) when an excitation is applied at node r. Since $\underset{\sim}{H}$ has the block nonzero pattern, we can see that the derivative of f_{qr} w.r.t. x_{ℓ} is not equal to zero only if the node q is a node of the subnetwork N_{α}.

$$\frac{\partial f_{qr}}{\partial x_{ij}} = \begin{cases} (h_{qi}-h_{qj})(v_{ir}-v_{jr}) & \text{when node } q \in S_{\alpha} \\ 0 & \text{otherwise} \end{cases}. \tag{22}$$

Jacobian matrix $\underset{\sim}{J}$ is a block matrix of the size mxp (m is the number of external nodes which can be used to measure voltages and p is the number of elements in the network). When the external nodes and all the elements are numbered in the order of subnetworks in which they are included then the Jacobian matrix has the structure as shown below:

$$\begin{array}{c} \quad S_1 \qquad\qquad S_2 \qquad \cdots \qquad S_{\alpha} \qquad\qquad S_k \\ \quad 1\ 2..\ e_1;\ 1\ 2..\ e_2;\ \ldots;\ 1\ 2..\ e_{\alpha};\ldots;\ 1\ 2..\ e_k \\ \begin{array}{c} m_1 \\ c_{12} \\ m_2 \\ \vdots \\ m_{\alpha} \\ \vdots \\ m_k \end{array} \left[\begin{array}{cccccc} \boxed{} & & & & & \\ & \boxed{} & & & & \\ & & \ddots & & & \\ & & & \boxed{} & & \\ & & & & \ddots & \\ & & & & & \boxed{} \end{array}\right] \end{array}$$

IV. QR FACTORIZATION OF SENSITIVITY MATRIX.

The optimum set of test points should be selected to minimize the variances of the response prediction and the variances of the parameter evaluation. An efficient approach to test point selection is based on the QR factorization (QRF) of the system sensitivity matrix [4].

The QRF is primarily used as a robust linear systems solving algorithm [6]. The results reported in [4] show that the QRF process with pivoting provides a powerful technique for the test points selection, the estimation of prediction variances, and the element testability. To maximize the determinant of the selected submatrix of the Jacobian we evaluate the product of diagonal elements of the R matrix.

In the QRF with pivoting we choose the row of the largest norm and orthogonalize all remaining rows to the selected one. Then the row of the largest norm of those remaining is selected and the orthogonalization step is repeated. The process continues until the norms of all the remaining rows are less then a preset threshold. The vectors selected during the QRF process correspond to the test points at which the actual measurements will be made.

In the KCL based method, all the partition voltages must be known (at least in one subnetwork) to formulate even a single test equation. Therefore, we modify the QRF process. In each step instead of selecting a vector with the largest norm we select the group of vectors. The QR factorization is run on the selected rows of the Jacobian to evaluate the product of the diagonal elements of the R matrix. Partial products are evaluated only for these elements of R which correspond to the selected rows. A group with the largest partial product is our pivot. The remaining rows in the Jacobian are orthogonalized w.r.t. the pivot rows.

It may happen that not all rows in the selected group are independent. This will be indicated by small values on the diagonal positions in R that correspond to the dependent vectors. Using a user defined threshold we eliminate such vectors from the selected group. Since different groups may have different number of the selected vectors we construct a vector of partial products as follows:

$$R_{pp} = \left[r_u \right] = \left[\prod_{k=1}^{u} R_{i_k} \right] \quad u=1,2,...g_i, \tag{23}$$

where R_{i_k} represents k–th diagonal element of R matrix in the ith group, and g_i is the number of elements in this group which are larger than the threshold value. Comparing two groups (say ith and mth) we first select

$$g = \min(g_i, g_m)$$

and choose the one with larger $r_g \in R_{pp}$.

The computer simulation shows that this selection reduces the number of excitations (and frequencies) with a small decrease in the accuracy of the solution (see Table 1).

V. RESULTS OF THE COMPUTER SIMULATIONS

As an example, consider the circuit of Fig. 2. Voltage measurements are taken at the nodes 1, 2, 4, 5 and 6. The circuit is decomposed at the measurement nodes into three subnetworks as shown in Fig. 3. The candidate set of test frequencies consists of 31 frequencies equally distributed in the log scale, giving 155 possible test measurements.

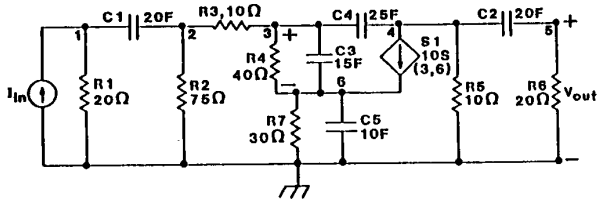

Fig. 2 An active circuit example (after [4]).

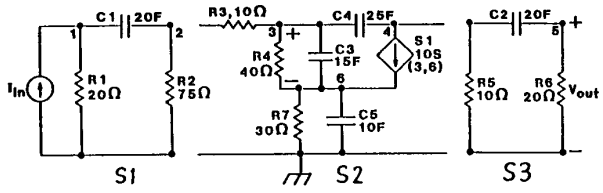

Fig. 3 Network decomposition of the example circuit.

The system of test equations is formed based on the KCL equations at the measurement nodes. The corresponding Jacobian matrix has the block structure as shown in Fig. 4.

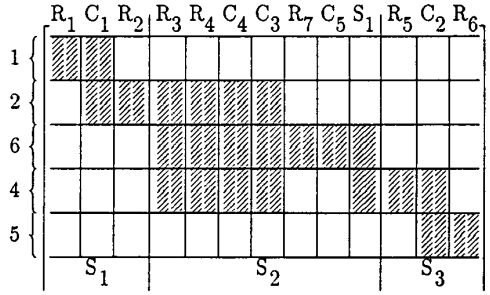

Fig. 4. The nonzero pattern of J.

The nominal element values are listed in Fig. 2. Assume that the relative element deviations are within 7% (refer to [4]). Denote ΔD to be the accuracy of the solution which represents the difference between the calculated and true element deviation. The element testability factors t_i are calcultaed as the ratio σ/σ_i, where σ and σ_i are the standard deviations of the measurement errors and the calculated change in the element value. σ_i are estimated from the element covariance matrix obtained during the QRF process.

Accuracy of the solution and testability factors obtained in the single and the group test selection are compared in Table 1. The single test selection uses orthogonalized vectors with the largest norm while the group test selection uses the partial products (23).

VI. CONCLUSIONS

A new method to test and calibrate linear circuits is proposed. As demonstrated the Jacobian matrix of the proposed test functions (8) is a sparse matrix with a very regular block structure. The computational speed can be increased and the memory space can be decreased when the sparse matrix technique is used. The system calibration and fault diagnosis can be performed with the measurements taken at the preselected test points.

Table 1. Accuracy of solution and Testability factors in the single and group test selections.

	Accuracy of solution ΔD		Testability factor	
	single	group	single	group
C4	0.00026	0.00068	0.1692	0.05134
C1	0.0	0.0	0.4381	0.1524
R1	-0.00002	-0.00002	0.7245	0.1602
R3	-0.00049	-0.00094	0.1664	0.0503
S1	-0.00339	-0.00267	0.04766	0.0354
C3	0.00037	0.00074	0.1178	0.05005
C2	-0.01011	-0.00539	0.00788	0.00721
R7	-0.00660	-0.00847	0.02678	0.02251
C5	-0.00523	-0.00427	0.03193	0.02737
R5	0.00626	0.00805	0.01915	0.01579
R4	0.10454	0.10399	0.01504	0.01759
R2	0.00074	0.00406	0.02743	0.00767
R6	0.00946	0.00463	0.00723	0.00698

The QR algorithm can be modified to select groups of test points, which correspond to all the voltage measurements. The factorization process can be organized to preserve the sparsity of the system Jacobian matrix. In this way a suboptimal selection of the test points can be performed. Its purpose is to minimize an effect of the measurement and roundoff errors on the accuracy of the parameters evaluation. In the case of the functional testing the test point selection minimizes the prediction standard deviations (see [4]).

Computer simulation confirmed feasibility of the proposed method in terms of the rank of the Jacobian matrix and obtained accuracy.

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