

# A DECOMPOSITION APPROACH FOR PARAMETER IDENTIFICATION IN LARGE SCALE NETWORKS

Hong Dai and Janusz A. Starzyk\*

Electrical Engineering Department, Lafayette College, Easton, PA 18042

\*Electrical and Computer Engineering Department, Ohio University, Athens, OH 45701

## Abstract

This paper presents an efficient parameter identification method for large scale networks based on the circuit decomposition technique. Parameter identification technique has wide applications in circuit modeling, fault diagnosis, testing and calibration. Its implementation, based on the sensitivity approach, is very useful in practice. However, it cannot handle large scale circuits because the sensitivity matrix is dense, which requires an enormous amount of memory space to store and takes much time to compute when the circuit size is large. To overcome these deficiencies was the main motivation behind this paper. A new developed method based on the circuit decomposition is presented. First, we present an organization of this method, its basic features and its algorithm. Then computer results for comparison of this method with a conventional, sensitivity based technique are given. Advantages of the new method are summarized in the conclusion.

## 1. Introduction

Parameter identification is very important for circuit modeling, fault diagnosis, testing and calibration. For example, when a new circuit is built, the responses of the circuit to one or several specific input signals must be measured to determine whether it works or not. If the responses are out of the range of design specifications, the circuit can be adjusted by calibrating its parameter values. In order to perform trimming, alignment or calibration, the actual parameter deviations must be estimated. The purpose of parameter identification is to find the actual parameter deviations using voltage measurements.

In general, parameter identification is more difficult than system analysis. Equations for determining parameter values from measurement data such as input and output voltages are nonlinear, even for a linear circuit. One numerical approach to parameter identification is to linearize these equations. A solution of the linearized equations can be obtained by the Newton-Raphson iteration process. This approach is called the first order approximation method.

The sensitivity matrix method, one of the first order approximation methods, is currently the most popular parameter identification technique. The sensitivity matrix, which contains derivatives of the responses with respect to all parameters, is evaluated based on the nominal parameter values  $p_0$ . The parameter deviations  $\Delta p$  are found through deviations of the responses  $\Delta v$  and the sensitivity matrix  $S$ , i.e.

$$S \Delta p = \Delta v \quad (1)$$

Once the parameter deviations  $\Delta p$  have been determined, the real parameter values  $p$  can be estimated by adding the deviations  $\Delta p$  to the nominal values  $p_0$ , i.e.

$$p = p_0 + \Delta p \quad (2)$$

The sensitivity matrix method can handle a broad category of networks and testing situations. Specialized formulas have been developed for this method when applied to linear and nonlinear networks or networks with reactive elements and switches. Different test equations are derived depending upon the type of measured responses such as time domain response, frequency response or harmonics of a periodic response [1,2].

Sensitivities are calculated in two steps: the network analysis and the sensitivity analysis. In the first step, equations of the "original network" are solved and the circuit responses are obtained. In the second step, equations of the "sensitivity network" are solved and the sensitivities of the circuit response are evaluated. The system matrix (Jacobian matrix) in the sensitivity analysis is the same as the system matrix at the convergence of iterations in the network analysis. Hence, the most important task is to evaluate the circuit response in the network analysis. Once the circuit response is evaluated, its sensitivities can be obtained by solving a system of linear equations.

However, the method shows some serious drawbacks when applied to large scale circuits. The first drawback is its low speed. In order to derive the sensitivity matrix, a circuit must be analyzed using simulators based on Newton's method, a sparse matrix technique, and numerical integration. Since the computation time in these simulators is very long for large circuits, the size of circuits that can be tested practically using the sensitivity matrix approach is limited to a few hundred elements.

Another drawback is the low accuracy of the sensitivity matrix method. In addition to errors caused by the first order approximation, the method is very sensitive to inaccuracies in the circuit model and in the numerical integration techniques, parasitics introduced by the test equipment and errors of time synchronization. Serious problems are associated with determination of the rank of the sensitivity matrix, testability factors, and ambiguity groups [3,4].

Finally, the sensitivity matrix method has large memory requirements, not only during the analysis but also at the solution of the test equations. Each transfer function is sensitive to variations of every network parameter. This causes the sensitivity matrix to be dense and makes numerical calculations expensive in the case of large networks.

To overcome these weak points of the sensitivity method we have developed a new method for parameter identification in large analog and mixed-mode circuits based on the decomposition approach.

## 2. Decomposition Approach

The idea of introducing a decomposition approach for parameter identification stemmed from similar approach for fault verification [5]. However, the form of equations obtained, and the processing steps required are different for the two approaches. In addition, the overall gain obtained by using the decomposition approach in the parameter identification is much greater than that in the fault verification technique.

Decomposition is a numerically effective way to solve large problems represented by a system of equations [6,7]. But its direct application to parameter identification problem is difficult. This difficulty is caused mainly by high density of test matrix formulated by the conventional method [4]. In this paper we first explain how to use decomposition to reduce complexity of the obtained test equations and show the corresponding test matrix is a sparse matrix. Then we present results of the computer simulated testing performed on networks of different sizes. We compare the results obtained from the decomposition method with those from the sensitivity matrix method.

In our approach, we assume that the measurements are taken at the selected nodes. Usually, selected nodes are limited to the external terminals or, in case of a large system, to the nodes between different subsystems or modules.

Removal of measurement nodes and adjacent branches decomposes the network into a number of weakly connected or isolated subnetworks. Analysis of these subnetworks can be performed independently from each other as the boundary conditions are known. This corresponds to analysis of subnetworks with known voltage sources connected at their terminals. In this case, current flow between subnetworks is not needed to solve subnetwork equations.

Internal voltages and other unknown variables  $x$  can be obtained in each subnetwork by solving iteratively Newton-Raphson equations.

$$M_k^i \Delta x_k^i = -f_k^i \quad (3)$$

where the subscript  $k$  stands for the  $k$ th iteration, superscript  $i$  stands for the internal nodes, and  $\alpha$  stands for the  $\alpha$ th subnetwork.  $f_k^i$  is the vector of the system functions corresponding to the internal nodes of the  $\alpha$ th subnetwork,  $x_k^i$  is the vector of system variables corresponding to the internal nodes of the  $\alpha$ th subnetwork and  $M_k^i$  is the Jacobian matrix of system equations, calculated at the nominal parameter values  $p_0$  and the system variable values  $x_{k-1}^i$  at the previous iterative step.

Sensitivities of the internal voltages w.r.t. network parameters can be obtained by solving an equation similar to (3):

$$M^i s^i = -B^i \quad (4)$$

where  $s^i$  is the vector of sensitivities of system variables  $x^i$  w.r.t. the parameters  $p_\alpha$  inside of the  $\alpha$ th subnetwork and  $B^i$  is the vector of derivatives of system functions  $f^i$  w.r.t. parameters  $p_\alpha$  [8].

Since the voltages at the measurement points are known exactly, their derivatives w.r.t. parameters are zero, i.e.

$$s^m = \frac{\partial x^m}{\partial p} = 0 \quad (5)$$

From equations (3) and (4) we can see that the internal variables  $x^i$  and their sensitivities  $s^i$  are computed at the subnetwork level which reduces computing time and increases the accuracy of the analysis results.

Solution obtained from (3) yields internal variables in ideal case of all parameters having their nominal values. If in real circuit parameters deviate from nominal values by  $\Delta p$ , the internal variables will change. This will cause changes in the external currents in each subnetwork. If we put these subnetworks back to obtain the original network, the sum of currents at the partition points may be different from zero. Therefore system equations at the partition points will not be satisfied.

In order to find parameter deviations, we differentiate system equations corresponding to measurement nodes w.r.t. parameters. Obtained matrix of derivatives will be denoted by  $T$  and called the test matrix.

$$T = \frac{d f^m}{d p} \quad (6)$$

where  $f^m$  is the vector of system functions at the measurement nodes. Matrix  $T$  can be evaluated by differentiating system functions  $f^m$  in each subnetwork and adding the obtained submatrices. (see [8] for more detailed discussion). Finally deviations in network parameters  $\Delta p$  can be obtained by solving the test equation

$$T \Delta p = -f^m \quad (7)$$

Numerical accuracy of the solution for  $\Delta p$  depends on the eigenvalues of the test matrix  $T$ . In order to reduce the condition number (determined by the ratio of largest and smallest eigenvalues) of the test matrix, we select test nodes, time instances, excitation levels and other controllable factors of test before performing the real measurements. Selection of test points, which improves numerical accuracy of test equations was discussed earlier by Stenbakken and Souders in [3].

In order to determine test points in advance, we first simulate the nominal network and formulate test matrices of its subnetworks. Then test point selection is performed using modified QR factorization on the test matrix. We check whether the rank of the test matrix is sufficient for parameter evaluation. This procedure may not always result in identification of all parameter values due to existence of ambiguity groups as reported in [4].

After test point selection, the real circuit is measured at the selected points and its parameters identified as described in this section. The flow diagram of a test procedure for parameter identification using the presented methods is shown in Fig. 1.

### 3. Computer Simulation Results

The procedure described above has been performed on circuits with different number of elements. The basic test circuit is built with a transistor amplifier cascaded by a low pass filter. It has 23 elements and 6 test nodes 1, 2, 3, 4, 5, 9. In the decomposition approach, the circuit is decomposed into 4 subcircuits as shown in Fig. 2. Bigger test circuits are obtained by combining 2, 4, 8 and 16 basic test circuits together with total number of 46, 92, 184 and 368 elements, respectively.

For each tested circuit, voltage measurements are taken at the test nodes. Time domain responses within the time interval 0 – 200 s are sampled. (The circuits are normalized). Then the parameter values are evaluated using the sensitivity method and the decomposition method respectively. The results obtained by these two methods are compared. For both methods, the computations were implemented in two stages: simulation stage and test stage.

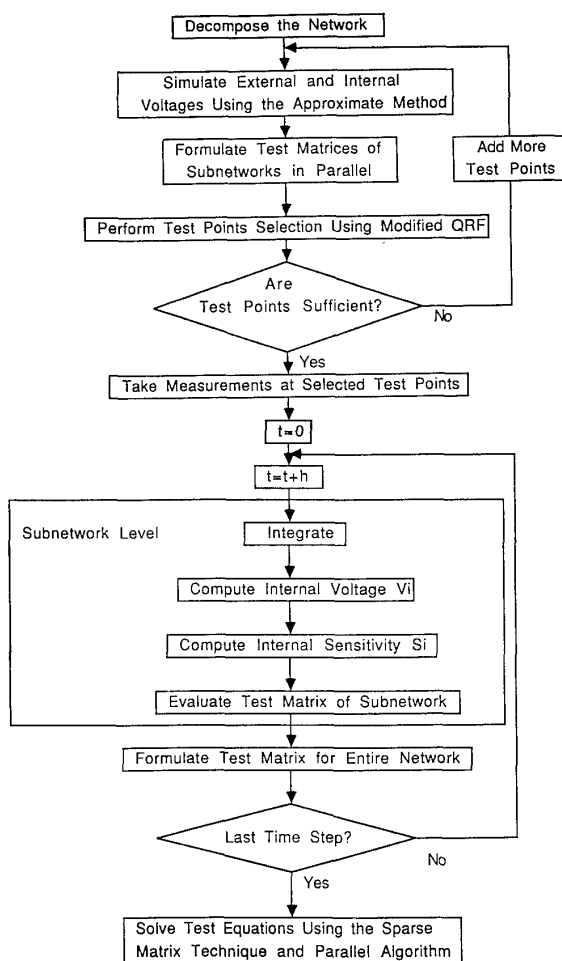


Fig. 1 Flow diagram of test procedure.

Using the sensitivity method, the sensitivity matrix which contains sensitivities of waveforms at measurement nodes w.r.t. to all elements in the circuit, have to be evaluated at the simulation stage first. Then the system equations (1) must be solved at the test stage to obtain the parameter deviations  $\Delta p$ .

Using the decomposition approach, the circuit can be decomposed into a number of subcircuits. The subcircuits can be further decomposed into a number of sub-subcircuits. At the simulation stage, voltages at the internal nodes of each subcircuit and their sensitivities to parameters inside the subcircuit are computed first. Then the test equations for the whole circuit are formulated. At the test stage, parameter deviations are obtained by solving these test equations. Since the coefficient matrix  $T$  of the test equation is sparse, the sparse matrix technique is used to speed up the computations [9].

In order to be able to compare both methods on the same basis we wrote a simple circuit simulator. Its overall performance is lower than that of the state of art analysis software, but the relative results indicating savings in computer time and memory space obtained by the new method are preserved.

The computations were implemented on SUN 3/280 at the Ohio University. The comparison of parameter deviations obtained by both methods is given in Table 1. In Table 1  $D$  stands for the true relative deviations from nominal values in percent,  $D_s$  and  $D_d$  represent similar deviations evaluated by the sensitivity and the decomposition methods respectively.

Table 1. Parameter deviations (in percent)

		$D$	$D_s$	$D_d$
1	R1	0.50	0.47	0.48
2	C1	-5.00	-5.24	-5.00
3	R2	2.50	2.89	2.66
4	R3	2.00	1.98	1.94
5	R4	6.67	0.04	1.83
6	C4	-4.00	-4.10	-3.96
7	R7	-2.00	-1.65	-2.15
8	C3	-6.67	-6.49	-6.69
9	C5	-5.00	-5.04	-4.97
10	R5	-2.00	-2.88	-2.04
11	C2	-5.00	-5.64	-4.98
12	R6	3.00	3.83	2.90
13	S1	-5.00	-4.76	-4.99
14	C6	-2.00	-2.94	-1.99
15	C7	-8.00	-8.17	-7.99
16	L1	2.00	2.28	1.95
17	C8	-4.00	-4.63	-3.99
18	C9	-4.00	-4.73	-3.99
19	L2	2.00	2.13	1.95
20	C10	-1.33	-1.51	-1.32
21	C11	-2.00	-2.30	-1.99
22	L3	2.66	2.89	2.59
23	C12	-1.00	-1.16	-0.99

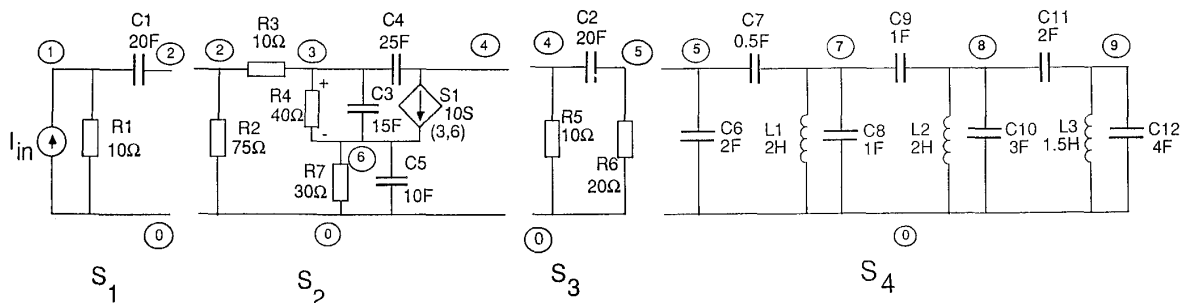


Fig. 2 Basic test circuit decomposed into 4 subcircuits.

In both techniques only parameter R4 was evaluated with poor accuracy, but this element has low testability factor as observed earlier [3].

The comparison of CPU time used during the circuit simulation and the test stages are shown in Figs. 3 and 4 respectively. Computational time increases almost linearly with the number of elements for the decomposition method, but polynomially for the sensitivity matrix method. In addition, as we compare cpu time used at the simulation stage and the test stage for the sensitivity matrix method, it is clear that the simulation time has become a major cost factor of the element identification process for circuits with number of elements large than 92. Whereas in decomposition method, the simulation time has been kept well below the test time. Besides that, the accuracy obtained by the decomposition method is higher than that obtained by the sensitivity matrix method.

#### 4. Conclusions

We have developed an efficient method for parameter identification in large scale networks. This method uses decomposition of the system equations by taking voltage measurements at the partition nodes. As a result, circuit analysis can be simplified and the results obtained by this method are more accurate than those by the conventional method. Measured voltages are used first to estimate internal voltages and then to formulate test equations.

Test equations can be prepared on the subnetworks level and the resulting test matrix is sparse. By solving test equations, we can evaluate all network parameters. We have compared test results of the sensitivity and the decomposition methods. The later is more efficient in terms of memory storage requirements and computing time than the former one. It also gives more accurate estimate for parameter values.

The decomposition method can be applied to test mixed-mode circuits. In this case, digital subcircuits will be tested using algorithms for digital testing while analog subcircuits will use test equations discussed in this paper.

Some designs may incorporate the original circuit with an additional circuitry to facilitate testing. This design practice, commonly used in digital circuit design, may also benefit complex analog circuits, such as artificial neural networks. Using multiplexers we may gain access to internal nodes of the circuit, increasing system observability.

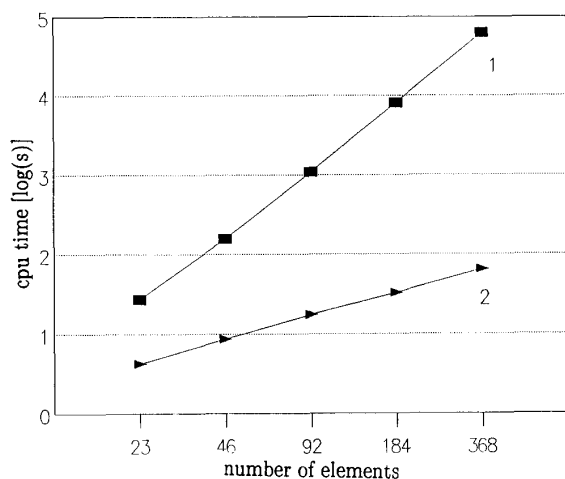


Fig. 3 CPU time used at the circuit simulation stage  
(1) sensitivity matrix method (2) decomposition method

#### Acknowledgements

The authors wish to thank Wu Xiaoming for his help in computer simulation and to acknowledge support for this work by the National Institute of Standards and Technology, U.S. Department of Commerce, under Grant No. 70NANBGH0662.

#### References

- [1] V. Visvanathan and A. Sangiovanni-Vincentelli, "A computational approach for diagnosability of dynamical circuits," *IEEE Trans. Computer-Aided Des.*, vol. CAD-3, 1984, pp. 165-171.
- [2] W. Nye, D. Riley, A. Sangiovanni-Vincentelli, and A.L.Tits, "DELIGHT.SPICE: An optimization - based system for the design of integrated circuits," *IEEE Trans. Computer - Aided Des.*, vol. CAD-7, 1988, pp. 501-519.
- [3] G. N. Stenbakken and T. M. Souders, "Test point selection and testability measures via QR Factorization of Linear Models," *IEEE Trans. Instrum. Meas.*, vol. IM-36, no. 2, June 1987, pp. 406-410.
- [4] H. Dai and T. M. Souders, "Time domain testing strategies and fault diagnosis of nonlinear analog circuits". To appear in *IEEE Trans. on Instrum. Meas.*, Feb., 1990.
- [5] A.E.Salama, J.A.Starzyk and J.W.Bandler, "A Unified decomposition approach for fault location in large analog circuits," *IEEE Trans. Circuits Syst.*, vol. CAS-31, 1984, pp.609-622.
- [6] J. A. Starzyk, "Signal-flow-graph analysis by decomposition method", *IEE Proc.*, vol. 127, Pt.G, no.2, pp. 81-86, 1980.
- [7] M. Vlach, "LU decomposition and forward-backward substitution of recursive bordered block diagonal matrices," *IEE Proc.*, vol. 132, Pt. G, no. 1, pp. 24-31, 1985.
- [8] J. A. Starzyk and H. Dai, "Time domain testing of large nonlinear circuits" *Proc. European Conf. on Circuit Theory and Design*, (Brighton, United Kingdom, 1989), pp. 390-394.
- [9] C.C. Paige and M. A. Saunders, "LSQR, an algorithm for sparse linear equations and sparse least squares," *ACM Trans. Math. Software.*, vol. 8, pp. 43-71, 1982.

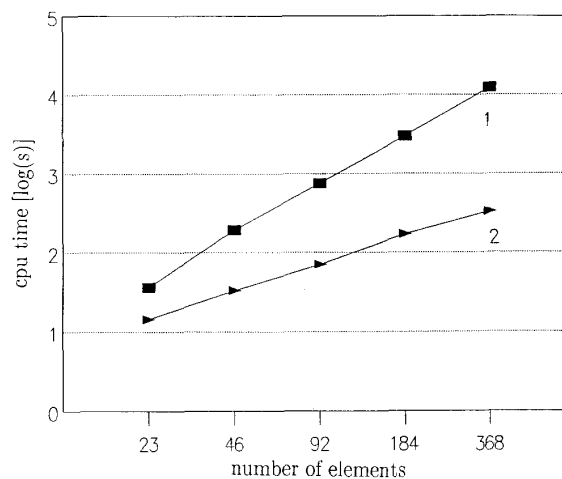


Fig. 4 CPU time used at the test stage  
(1) sensitivity matrix method (2) decomposition method