# Application of Gauss's Law

- At a point $x$, the electric field can be found as the charge enclosed, divided by the permittivity of the material ...

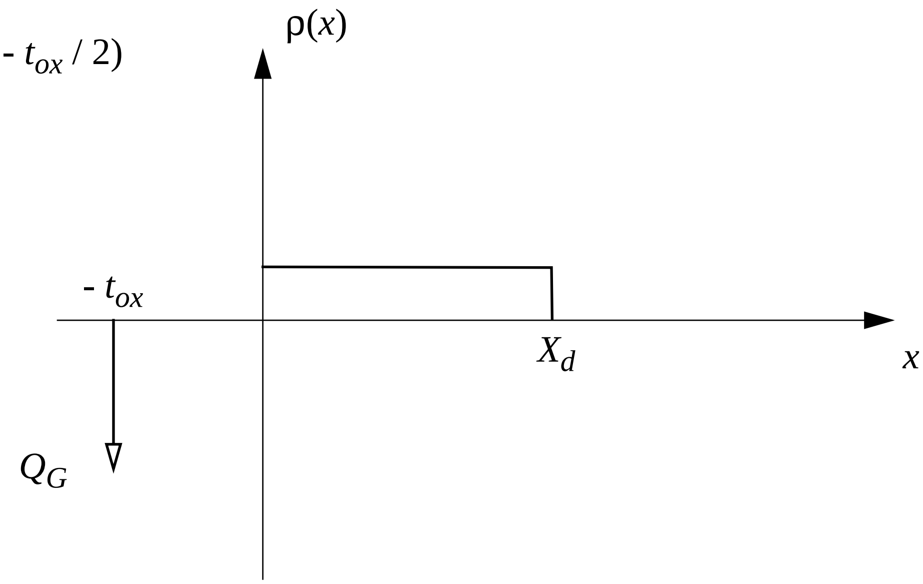
  *caveats* (warnings):

  (i) the field must be zero at the other side of the charged region

  (ii) the sign of the field can be found by keeping track of the $+x$ direction and the one-dimensional equivalent of the "outward normal;" however, the best approach is to know the sign of the field from the distribution of charge in the problem

- Example: metal-oxide-silicon structure

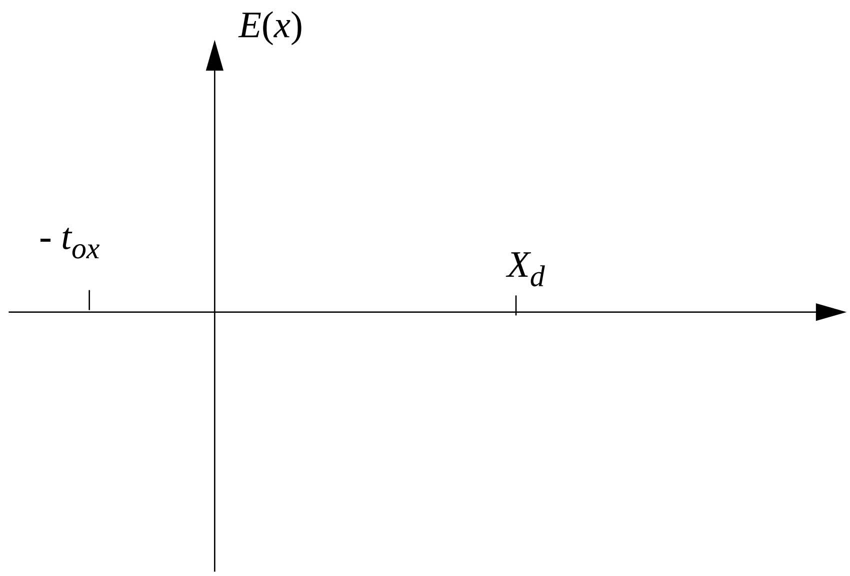
*Find* $E(x = -t_{ox} / 2)$



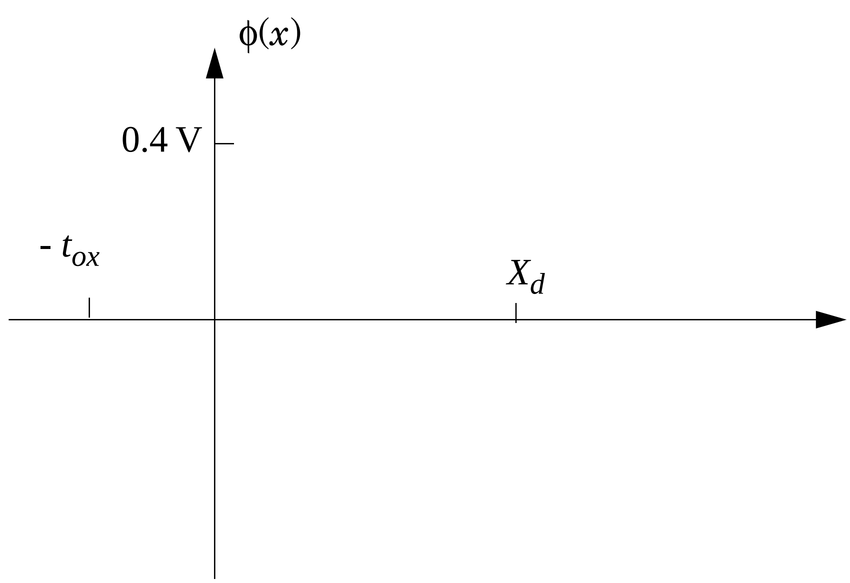

- Find $E(x = 0^{+})$ ... just inside the silicon

# Boundary Condition on $E$ (cont.)

- Sketch $E(x)$ from $x = - t_{ox}$ to $x = X_d$




- Sketch $\phi(x)$ through the structure, given that $\phi(X_d)$ = 400 mV



# Potential and Carrier Concentration in Silicon

- What is a convenient reference for the electrostatic potential in silicon?

*Thermal equilibrium:* no external stimulus --> must have:

$J_p = 0$ and $J_n = 0$.

$$\therefore 0 = q n_o \mu_n E + q D_n \frac{dn_o}{dx} = q n_o \mu_n \left(-\frac{d\phi}{dx}\right) + q D_n \frac{dn_o}{dx}$$

$$d\phi = \frac{D_n}{\mu_n}\left(\frac{dn_o}{n_o}\right) = \frac{kT}{q}\left(\frac{dn_o}{n_o}\right) = V_{th}\left(\frac{dn_o}{n_o}\right)$$

where we have used Einstein's relation.

$$\frac{D_n}{\mu_n} = \frac{kT}{q} = V_{th} = 25 \text{ mV at "cool" room temperature to}$$

26 mV at "warm" room temperature

$V_{th}$ is called the *thermal voltage.*

# The Intrinsic Potential Reference

- By integrating the equation relating potential to the electron concentration from a position $x_a$ to position $x$, we find that:

$$\phi(x) - \phi(x_a) = V_{th} \ln\left(\frac{n_o(x)}{n_o(x_a)}\right)$$

We can choose any reference; one convenient choice is to set:

$$\phi(x_a) = 0 \text{ when } n_o(x_a) = n_i = 10^{10}\ \text{cm}^{-3} \text{ at room temperature.}$$

- Using this reference, the potential in thermal equilibrium can be found, given the electron concentration:

$$\phi = V_{th} \ln\left(\frac{n_o}{n_i}\right) = (26\text{mV}) \ln(10) \log\left(\frac{n_o}{10^{10}}\right) = (60\text{mV}) \log\frac{n_o}{10^{10}}$$

Donor concentrations from $10^{13}$ to $10^{19}$ cm$^{-3}$ therefore correspond to potentials of (60 mV) x 3 = 180 mV to (60 mV) x 9 = 540 mV (at room temperature)

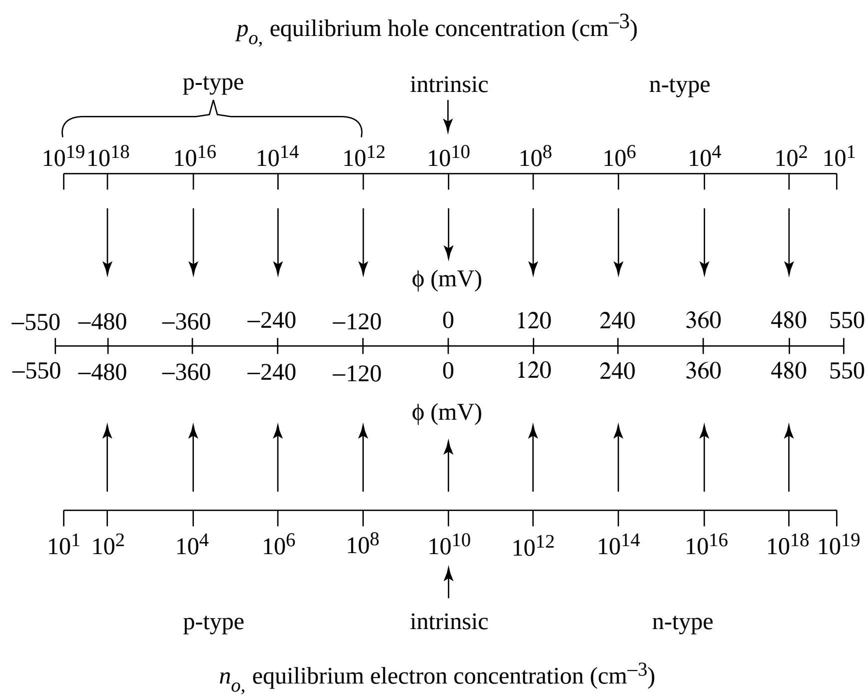
# The 60 mV Rule

The hole concentration can also be related to the potential, by repeating the derivation starting with $J_p = 0$ or by substituting

$$p_o = n_i^2 / n_o$$

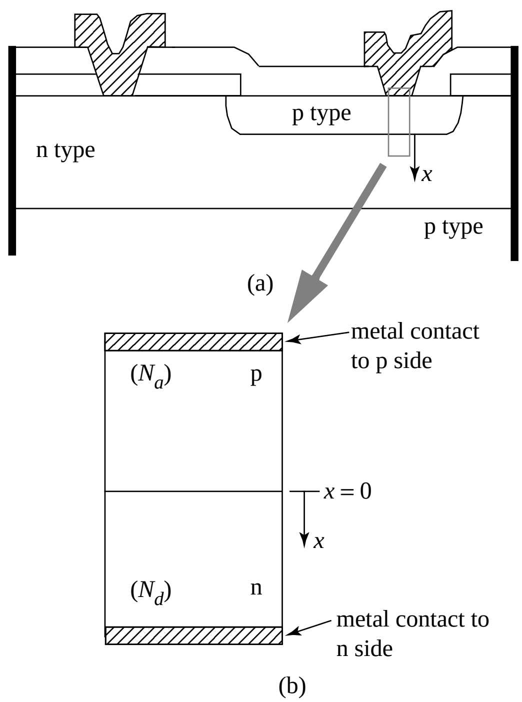
into the 60 mV rule for electrons. The result is:

$$\phi = V_{th} \ln\left(\frac{n_i}{p_o}\right) = (-26\text{mV})\ln(10)\log\left(\frac{p_o}{10^{10}}\right) = (-60\text{mV})\log\left(\frac{p_o}{10^{10}}\right)$$



# pn Junctions

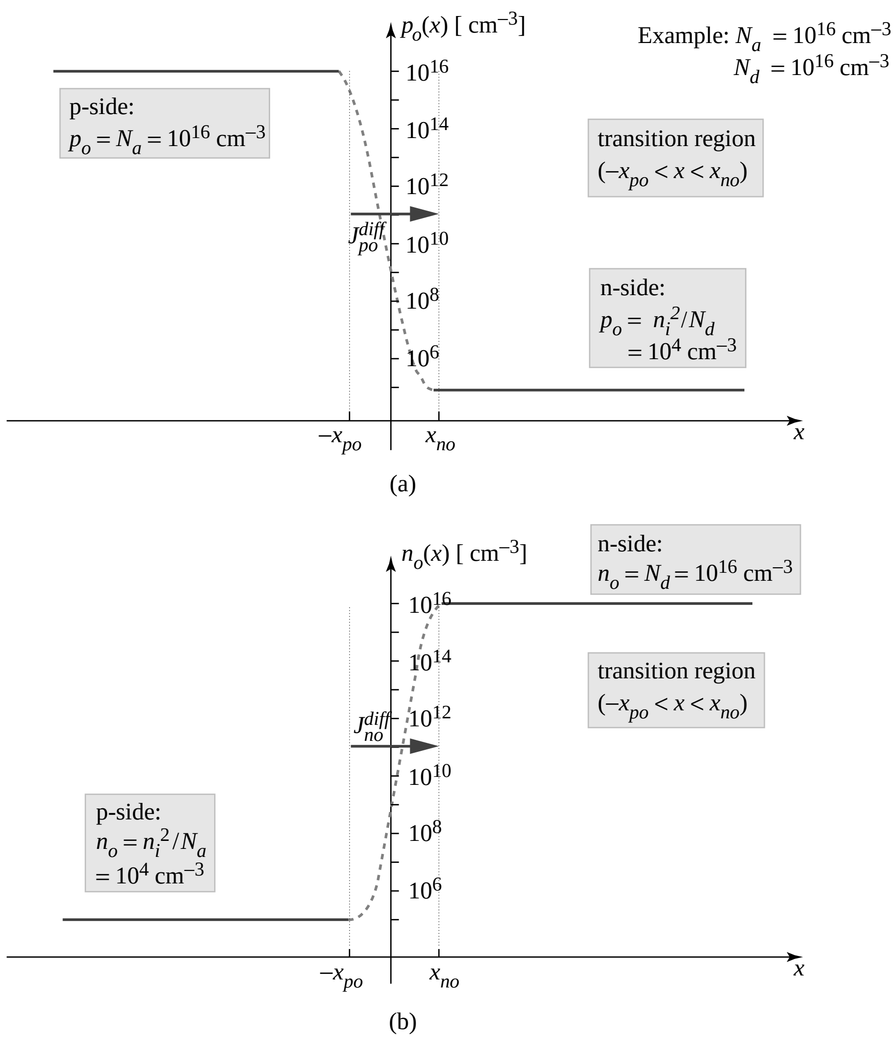
- ubiquitous IC structure -- pn junctions are everywhere!




- thermal equilibrium:  no hole current, no electron current ... no voltage applied between metal interconnects (could short them together)

# Diffusion Currents in Thermal Equilibrium

- huge gradients in hole and electron concentration --> assume a transition region between - $x_{po}$ and + $x_{no}$




note: we don't know how wide the transition region is (yet)

# Drift and Diffusion in the Transition Region

- $J_{no} = 0$ and $J_{po} = 0$ due to equilibrium

  --> negative electric field in the transition region is needed ...

  where do + and - charges come from?

- Answer: the roll-off in electron concentration between $x = 0$ and $x_{no}$ means that

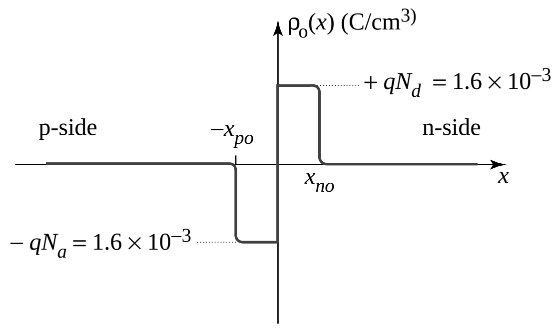
$$\rho_o(x) = q(-n_o(x) + N_d) > 0$$

since $n_o(x) < N_d$ on the n-side of the transition region

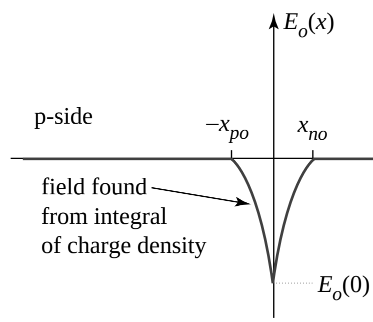
- On the p-side, the charge density is negative, since the hole concentration rolls off between $x = - x_{po}$ and $x = 0$.

# Qualitative Electrostatics in Equilibrium

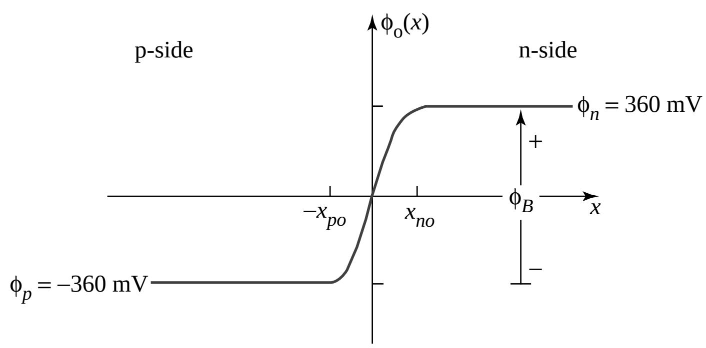
■ From the charge density, we can find the electric field and the potential




(a)




(b)




(c)

# Quantitative pn Junction in Thermal Equilibrium

## The Depletion Approximation

- In the bulk regions far away from the junction, we can approximate

$$\rho = 0$$

- Near the junction, the charge density is non-zero. For example, on the n-side of the junction in the transition region, $0 < x < x_{no}$:

$$\rho = q\,(p_o + N_d - n_o - N_a) = q\,(N_d - n_o)$$

since there are no acceptors on this side ($N_a = 0$) and the hole concentration is negligible --> $p_o = 0$ (approx.)

The maximum positive value for charge density on the n-side is when there are *no electrons present in equilibrium* -- that is, when the silicon in the transition region is *depleted* of electrons.

For hand calculations, we will assume that

$$\rho = \rho_{max} = q\,N_d \quad ( 0 < x < x_{no} )$$

$$\rho = -\,q\,N_d \quad ( -x_p < x < 0)$$

and proceed to find the width of the transition region, which we will rename the *depletion region*. The charge density is assumed to fall off *abruptly* from these values to zero in the bulk regions, where $x < -\,x_{po}$ and $x > x_{no}$

# One more time ...

- Bulk silicon is *NEUTRAL*, to a good approximation

  >> region 1 is *bulk*:

$$\rho = q(N_d + p_o - N_a - n_o) \cong 0 \quad \text{-->} \quad p_o \cong N_a$$

  >> region 4 is *bulk*:

$$\rho = q(N_d + p_o - N_a - n_o) \cong 0 \quad \text{-->} \quad n_o \cong N_d$$

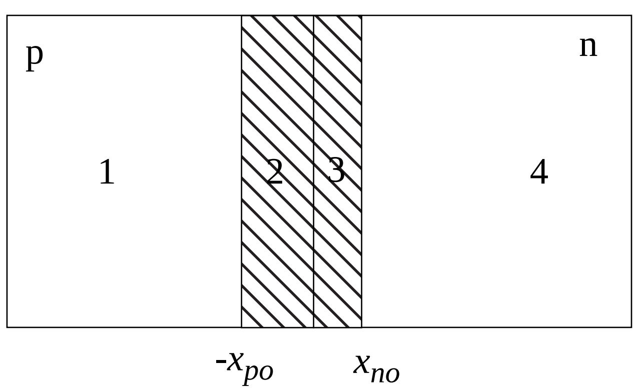
- Near the junction, the silicon is *DEPLETED* of mobile carriers:

  >> region 2 is *depleted*:

$$\rho = q(N_d + p_o - N_a - n_o) \cong -qN_a$$

  >> region 3 is *depleted*:

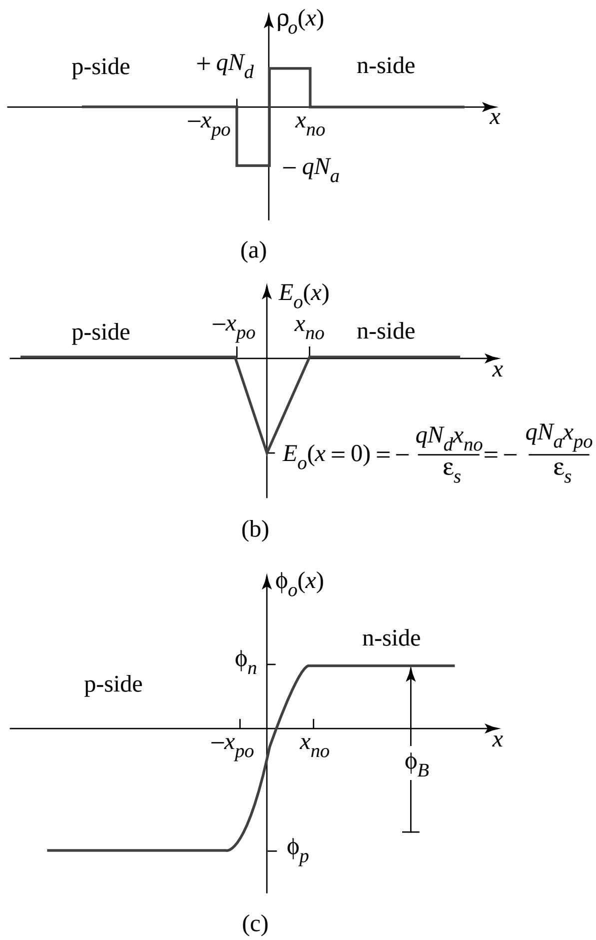
$$\rho = q(N_d + p_o - N_a - n_o) \cong qN_d$$




$x_{no}$, $x_{po}$ are not known yet -- use boundary conditions to find them

# pn Junction in Thermal Equilibrium:

## Using the Depletion Approximation




- For detailed calculations, see Section 3.4. Analysis is straightforward, but involved. Use the fact that:

  > Charge in depletion region must sum to zero (why?)

  > Electrostatic potential is continuous

# Depletion Widths in Thermal Equilibrium

$$x_{po} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_a}\right)\left(\frac{N_d}{N_d + N_a}\right)}$$

$$x_{no} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{qN_d}\right)\left(\frac{N_a}{N_d + N_a}\right)}$$

$$X_{do} = x_{no} + x_{po} = \sqrt{\left(\frac{2\varepsilon_s\phi_B}{q}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

- Asymmetric junctions: i.e., $N_a >> N_d$ or $N_d >> N_a$.

  >> most of depletion width is on the side with the *lower* doping, since

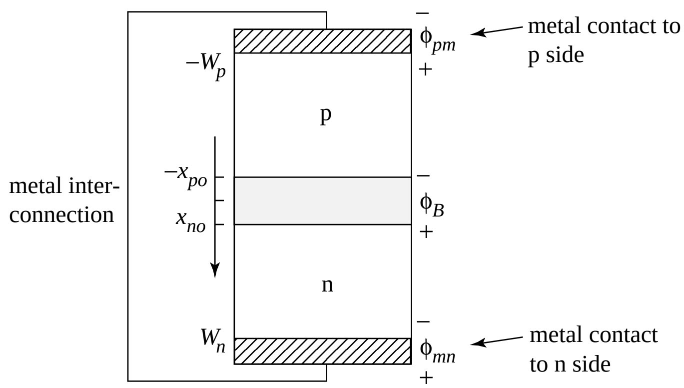
$$\frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_d} \qquad (N_a \gg N_d)$$

$$\frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_a} \qquad (N_d \gg N_a)$$

  >> most IC pn junctions are highly asymmetric

# pn Junction under Reverse Bias

- First, we must understand the *complete* structure of the pn junction-- starting in thermal equilibrium:




- How can $V_D = 0$ and the built-in potential barrier be $\phi_B = 1$ V (approx.)?

  Answer: look at the complete circuit ... including the potential barriers at the p-type silicon-to-metal ($\phi_{pm}$) and the metal-to-n-type silicon ($\phi_{mn}$) junctions.

- Kirchhoff's Voltage Law:

$$0 = \phi_{pm} + \phi_B + \phi_{mn}$$

therefore, the built-in voltage is given by:

$$\phi_B = -\phi_{pm} - \phi_{mn}$$