Linear Algebra and Optimization in Python

Lecture 01.a

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Python Programming Stack

- **Python** = object-oriented, interpreted, scripting language.
  - imperative programming, with functional programming features.

- **NumPy** = package for powerful N-dimensional arrays:
  - sophisticated (broadcasting) functions.
  - useful linear algebra, Fourier transform, and random number capabilities.

- **SciPy** = package for numerical integration and optimization.

- **Matplotlib** = comprehensive 2D plotting library.
import numpy as np

- np.array()
  - indexing, slices.
- ndarray.shape, .size, .ndim, .dtype, .T
- np.zeros(), np.ones(), np.arange(), np.eye()
  - dtype parameter.
  - tuple (shape) parameter.
- np.reshape(), np.ravel()
  - also np.ndarray.
- np.amax(), np.maximum(), np.sum(), np.mean(), np.std()
  - axis parameter, also np.ndarray
- np.stack(), np.[hv]stack(), np.column_stack(), np.split()
- np.exp(), np.log(),
NumPy: Broadcasting

• Broadcasting describes how numpy treats arrays with different shapes during arithmetic operations.
• The smaller array is “broadcast” across the larger array so that they have compatible shapes, subject to broadcasting rules:
  – NumPy compares their shapes element-wise.
  – It starts with the trailing dimensions, and works its way forward.
  – Two dimensions are compatible when:
    • they are equal, or one of them is 1.

- [https://docs.scipy.org/doc/numpy-dev/user/basics.broadcasting.html](https://docs.scipy.org/doc/numpy-dev/user/basics.broadcasting.html)
Other Numpy Functions

- `np.dot()`, `np.vdot()`  
  - also `np.ndarray`
- `np.outer()`, `np.inner()`

- `import numpy.random as random:`  
  - `randn()`, `randint()`, `uniform()`

- `import numpy.linalg as la:`  
  - `la.norm()`, `la.det()`, `la.matrix_rank()`, `np.trace()`  
  - `la.eig()`, `la.svd()`  
  - `la.qr()`, `la.cholesky()`

- [https://docs.scipy.org/doc/numpy/reference/routines.linalg.html](https://docs.scipy.org/doc/numpy/reference/routines.linalg.html)
Implementation: Vectorization

- **Version 1**: Compute gradient component-wise.

\[
\nabla E(w) = \sum_{n=1}^{N} (h_n - t_n) x_n^T
\]

```python
grad = np.zeros(K)
for n in range(N):
    h = sigmoid(w.dot(X[:, n]))
    temp = h - t[n]
    for k in range(K):
        grad(k) = grad(k) + temp * X[k, n]
```

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Implementation: Vectorization

- **Version 2**: Compute gradient, partially vectorized.

\[
\nabla E(w) = \sum_{n=1}^{N} (h_n - t_n)x_n^T
\]

\[
grad = np.zeros(K)
\]

for n in range(N):
    \[
grad = grad + (\text{sigmoid}(w \cdot X[:, n]) - t[n]) \times X[:, n]
\]
Implementation: Vectorization

- **Version 3**: Compute gradient, vectorized.

\[
\nabla E(w) = \sum_{n=1}^{N} (h_n - t_n)x_n^T
\]

\[
\text{grad} = X\cdot\text{dot}\left(\text{sigmoid}(w\cdot\text{dot}(X)) - t\right)
\]

```python
def sigmoid(x):
    return 1 / (1 + np.exp(-x))
```

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import scipy

- scipy.sparse.coo_matrix()
  groundTruth = coo_matrix((np.ones(numCases, dtype = np.uint8),
    (labels, np.arange(numCases)))).toarray()

- scipy.optimize:
  - scipy.optimize.fmin_l_bfgs_b()
    theta, _, _ = fmin_l_bfgs_b(softmaxCost, theta,
      args = (numClasses, inputSize, decay, images, labels),
      maxiter = 100, disp = 1)
  - scipy.optimize.fmin_cg()
  - scipy.minimize

https://docs.scipy.org/doc/scipy-0.10.1/reference/tutorial/optimize.html